Innovations, Challenges, and Future Opportunities

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electromagnetic waves
Maxwell’s Equations

Gauss' Law of Electricity
\[ \int_{\text{surf}} \varepsilon E \cdot ds = \int_{\text{vol}} \rho^\text{free}_v \, dv \]
\[ \nabla \cdot E = \frac{1}{\varepsilon} \rho^\text{free}_v \]

Gauss' Law of Magnetism
\[ \int_{\text{surf}} H \cdot ds = 0 \]
\[ \nabla \cdot H = 0 \]

Faraday's Law of Induction
\[ \int_{\text{loop}} E \cdot dl = -\frac{d}{dt} \int_{\text{surf}} \mu H \cdot ds \]
\[ \nabla \times E = -\mu \frac{dH}{dt} \]

Ampere-Maxwell's Law
\[ \int_{\text{loop}} H \cdot dl = \int_{\text{surf}} J \cdot ds + \frac{d}{dt} \int_{\text{surf}} \varepsilon E \cdot ds \]
\[ \nabla \times H = \sigma E + \varepsilon \frac{dE}{dt} \]
Wave Equation

\[ \nabla^2 E = \mu^* \sigma \frac{\partial E}{\partial t} + \mu^* \varepsilon^* \frac{\partial^2 E}{\partial t^2} \]

Solution:
\[ E_y = E_0 e^{-\alpha \lambda} e^{j(\omega t - \kappa x)} = E_0 e^{j(\omega t - \gamma^* x)} \]

Then
\[ \gamma^* = \alpha + j \kappa = \sqrt{jo \sigma \mu^* - \omega^2 \varepsilon^* \mu^*} \]

Faraday:
\[ H_z = -j \frac{\gamma^*}{\mu \omega} E_y \]
Phase Velocity

\[ V_{ph} = \frac{\omega}{\text{Im} \ (\gamma^*)} = \frac{\omega}{\text{Im} \left( \sqrt{j\omega \sigma \mu^* - \omega^2 \varepsilon^* \mu^*} \right)} \]

*non-ferromagnetic / dielectric*
\[ \mu^* = \mu_0 \quad \varepsilon^* = \varepsilon' \quad \sigma = 0 \]

\[ V_{ph} = \frac{c_0}{\sqrt{\varepsilon'/\varepsilon_0}} \]

Attenuation

\[ \alpha = \text{Re} \ (\gamma^*) = \text{Re} \left( \sqrt{j\omega \sigma \mu^* - \omega^2 \varepsilon^* \mu^*} \right) \]

*non-ferromagnetic*
\[ \mu^* = \mu_0 \quad \varepsilon^* = \varepsilon' + j\varepsilon'' \quad \sigma \]

\[ \alpha = \frac{\omega \sqrt{\varepsilon'/\varepsilon_0}}{c_0} \sqrt{\frac{1}{2} \left( \sqrt{1 + \tan^2 \delta} - 1 \right)} \]
Skin depth

\[ S_d = \frac{1}{\alpha} = \frac{1}{\text{Re}(\gamma^*)} \]

Impedance

\[ z^* = \frac{E_y}{H_z} = j \frac{\omega}{\gamma^*} \mu^* \]

Reflection and Transmission

\[ R^* = \frac{1 - (z_1^*/z_2^*)}{1 + (z_1^*/z_2^*)} \]
\[ T^* = \frac{2}{1 + (z_1^*/z_2^*)} \]
Electromagnetic Parameters

Conductivity \( \sigma \)

Permittivity \( \varepsilon^* = \varepsilon' - j \varepsilon'' \)

Permeability \( \mu = \mu' - j \mu'' \)

Effective conductivity
\[
\sigma_{\text{eff}} = \varepsilon' \omega \mu_r'' + (\sigma + \varepsilon'' \omega) \mu_r'
\]
details and references in
Santamarina, Klein and Fam
Soils and Waves – J. Wiley

electromagnetic properties
Conductivity - Electrolytes

![Graph showing conductivity vs. concentration for NaOH, NaCl, and CaCl₂](image-url)
Bulk and Surface Conduction

\[ \sigma_{\text{soil}} = n \sigma_{\text{el}} + (1 - n) 2 \rho_p \lambda_{\text{ddl}} S_a \]

\[ \sigma_{\text{soil}} = \alpha n^\beta \sigma_{\text{el}} \] (Archie)
Conductivity: Archie?

\[
\sigma_{\text{soil}} = n \sigma_{\text{el}} + (1 - n) 2 \rho_g \lambda S_a
\]
Conductivity - Summary

Controlled by \((1-n) 2\rho \lambda S_s\)

\[\sigma_{\text{el}} = \sigma_{\text{soil}}\]

Controlled by \(n\sigma_{\text{el}}\)
Polarization – Single phase

Direction of Applied Field

<table>
<thead>
<tr>
<th>Electronic (resonance)</th>
<th>Ionic (resonance)</th>
<th>Orientational (relaxation)</th>
</tr>
</thead>
<tbody>
<tr>
<td>t = $10^{-16}$ s (Ultraviolet)</td>
<td>t = $10^{-13}$ s (Infrared)</td>
<td>t = $9 \times 10^{-12}$ s (Microwave – water)</td>
</tr>
</tbody>
</table>
Two-phase media - Spatial polarization

- (no relaxation)
- Maxwell relaxation
- Wagner relaxation
- Semi-permeable membrane
Double layer effects

Direction of Applied Field

Stern layer
- (Infrared)

Bound water (relaxation)
- (Radio frequency)

Double layer (deionized)

Double layer (electrolyte)

Double layer - Normal

Particle interactions
- (surface conduction)
Water-Ion Interaction

\[ f = 1.3 \text{ GHz} \]

![Graph showing the relationship between ionic concentration and \( \kappa' \) for different ions (CaCl\(_2\), NaCl, LiCl, FeCl\(_3\), KCl).]
Permeability

iron fillings in kaolinite – $f = 10$ kHz

$\mu'_{\text{rel}} = 1 + 4 v_{\text{Fe}} + 7 v_{\text{Fe}}$

$\mu'_{\text{rel}} = 1 + 3 v_{\text{Fe}}$

Maxwell

Wagner

volume fraction of iron filings

$\mu'_{\text{rel}}$ vs volume fraction of iron filings
Permeability

\[ \mu_{rel}^\pi \]

\[ \mu_{rel}'' \]

frequency [Hz]

frequency [Hz]

(a)    (b)    (c)    (d)    (e)    (f)    (g)
All data by Dante Fratta (U. Wisconsin)

TDR measurements
The Cable

corresponds to \(2L_{\text{cable}}\)

\[\Delta t\]

*the probe = complex end-reflector*  
*signal changed sign at equipment*
The Probe

corresponds to $2L_{\text{probe}}$

$f \sim 1 \text{ to } 3 \text{ GHz}$

dispersion

multiples
Short

where is zero-time?

composite reflection at top
open and short tip impedance
Short

where is zero-time?
composite reflection at top
open and short tip impedance
Boundaries: Normal Plate

does not see ahead of tip
Boundaries: Normal Plate

does not see ahead of tip
Boundaries: Parallel Plate

does not feel outside inter-rod?
Boundaries: Parallel Plate

H-field effect!
Permeability

Expect minor effect
Permeability

- 0.1% iron filings
- 5% iron filings
- 10% iron filings
- 15% iron filings
- 20% iron filings
- 25% iron filings

expect minor effect
Conductivity

- Dry sand
- Drained tap water
- Drained 0.2 M NaCl solution
- Tap water
- 0.2 M NaCl solution

Good assessment of conductivity
Conductivity

- Dry sand
- Tap water
- Drained tap water
- 0.2 M NaCl solution
- Drained 0.2 M NaCl solution

recall skin depth
may not see tip reflection
Heterogeneity – Layering
Varved Clay

X-Ray  Photograph  Needle probe measurements

Cho, Lee
Heterogeneity - Layering

In Air
disk at 29 cm - TIP
disk at 22.5 cm
disk at 15 cm
disk at 7.5 cm
disk at 0.0 cm - TOP
2 disks: 15 and 29 cm
2 disks: 7.5 and 22.5 cm
4 clay disks: 7.5, 15, 22.5, and 29 cm

Time / Δt

300 400 500 600 700 800

a clay seam may hide the rest
(very high mismatch in this case)
Heterogeneity in Water Content

more than one primary reflections
Insertion Effects

Undrained

Drained
Insertion: Volumetric Strain = f(void ratio)
Large vs. Small Particles

Gravel – $d_{50} = 20$ mm
Large vs. Small Particles

Gravel – $d_{50} = 20$ mm

higher local porosity in gravel

Brillouin LP filter?
Summary

NOT a simple scalar $f(\text{geometry, soil})$

Input

1\textsuperscript{st} reflection

2\textsuperscript{nd} reflection

3\textsuperscript{rd} reflection

multiples
Summary

The connection to probe:

- sequence of electrical and geometrical changes
- response is a function of the soil itself
- when is time zero? what signal gets to the soil?

Compare the 2\textsuperscript{nd} and 3\textsuperscript{rd} reflections (if 3\textsuperscript{rd} is not lost in noise)

Geometric dispersion + attenuation: signal widens

Ferromagnetism: expect small effect

Insertion effects and preferential packing (aggravated in coarse soils)

Complex signal: consider spatial variability

- multiple interpretations of multiples
- many unknowns $\rightarrow$ inversion may be ill defined

Information conservation $\rightarrow$ simple models (Ockham’s criterion)
process monitoring
Measurements
Sedimentation

- Suspension
- Soil

K. Klein
Pressure diffusion

(a)  

(b)  

\[ \kappa' \]  

\[ \kappa''_{\text{eff}} \]  

stress \([\text{kPa}]\)  

local volumetric water content  

DeLoo (Table 11.9)  

1.3 GHz  

0.20 GHz
Pressure diffusion

![Graph showing the relationship between time (in minutes) and shear wave velocity (in m/s) for different values of q (in kPa). The graph includes data points for q=305, 154, 76, 38, 19, and 0 kPa, with time ranging from 0.1 to 10,000 minutes and shear wave velocity ranging from 0 to 350 m/s.]
Cementation (bentonite-cement)

M. Fam
Cementation (sand-cement)

Conf. Press: 70 kPa
Conf. Press: 415 kPa

Constant confinement:
\( \sigma' = 415 \text{ kPa} \)
\( \sigma' = 70 \text{ kPa} \)

A. Fernandez
Gas hydrates

Kvenvolden and Lorenson, 2001
Real Permittivity (Kaolinite + THF + H₂O)

Temperature [°C]

Real permittivity $k'$

hydrate

ice

Time [min]

F. Francisca
Elastic waves

$V_p$ evolution

$V_s$ evolution

Temperature decrease

Temperature increase

Phase transf.

TS. Yun
Penetration-based Geophysical Systems

SV Source (Fernandez)

3D Geophone (Stokoe – UT)

Conductivity tip
Boundary measurements - Tomography
closing thoughts
Measurement $\kappa^* \sigma^* \mu^*$
TDR signature = input $\ast$ (geometry AND spatially varying material)

Better measurement interpretation
Inversion: caution… follow Ockham’s criterion

Inherent: insertion volume change
Consider non-intrusive implementation

Complementary information
  Electromagnetic & elastic waves
  Small perturbation & large-strain penetration testing

Wave parameters: relevant to engineering
Laboratory and field
Wide range of geotechnical processes

Boundary measurements: invert for internal conditions
Thank You