1 INTRODUCTION

Energy geotechnology involves geotechnical phenomena and processes related to energy, from resource recovery to infrastructure and waste management. Energy resources include fossil fuels (90% of all primary sources - coal, petroleum, and gas), nuclear, hydroelectric, and other renewable sources (wind, geothermal, solar, tidal, biomas). The most critical energy-related waste storages include: CO2 geological storage (from fossil fuels), fly ash (from coal), nuclear waste, and coal-mining waste.

Resource recovery, energy infrastructure and waste management often involve multi-phase fluid conditions (Table 1 – classical infrastructure related conditions are not addressed in this manuscript). The most relevant cases are:

- L-G: water-air, water-CO2 and water-methane interfaces (as well as other biogenic and thermogenic gases). The liquid L has molecules of the gas in solution, and the gas contains molecules of the liquid.
- L1-L2: water-liquid CO2 (geological C-storage), and water-oil (petroleum reservoirs). Both liquids include molecules of the other liquid in solution.
- L-I: water-ice and water-hydrate. Related analyses can often be interpreted as the “solid” ice or hydrate phase behaving as a high viscosity fluid.

The purpose of this manuscript is to extend fundamental concepts in unsaturated soil behavior to address mixed-fluid conditions in energy geotechnol-
the other. The average residing time for a molecule at the surface of a liquid is in the order of $10^{\sim} \text{s}$ (Shaw 1992).

Interfacial phenomena are understood at the atomic level. The van der Waals force is the result of Coulombian interactions (Fig. 1a) between atoms or molecules that experience instantaneous polarization. A dipole experiences orientation polarization next to an ion (Fig. 1b); this situation is extended to the mutually induced rotation of two interacting dipoles (Keesom – Fig. 1c). An atom experiences displacement of the electron cloud relative to the nucleus in the vicinity of a charge or ion (Fig. 1d). Furthermore, a dipole may polarize an otherwise non polar molecule (Debye – Fig. 1e), and two non polar molecules may become mutually polarized because the instantaneous position of electrons confers an atom or molecule with some instantaneous polarity even when the time-averaged polarity is zero (London dispersion – Fig. 1f). The resultant Coulombian force for the instantaneous charge configuration shows mutual attraction in all these cases. The van der Waals force includes Debye, Keesom and London contributions (Fig. 1c, 1e, & 1f). The atomic analysis of van der Waals interactions helps explain surface tension and contact angle.

\[ T_s \] [mN/m]. This situation applies to L-G, L₁-L₂ and L-I interfaces.

![Figure 1](image1.png)

**Figure 1. Electrical forces between ions and molecules.** The van der Waals force combines Debye, Keesom and London contributions (Santamarina et al. 2001).

2.1 Surface tension

The time-average van der Waals attraction is isotropic when the molecule is away from the boundary. However, induced polarizations become anisotropic for molecules along the boundary and higher van der Waals attraction develops along the interface (Fig. 2). This situation alters the molecular organization in fluids near the interfaces. The altered molecular arrangement extends for about 5-to-10 mono-layers away from the interface. For example, water molecules at the water-vapor interface prefer to be oriented with their negative side towards the vapor phase, while random bulk conditions are attained at a distance of 1~2nm from the interface (Butt et al. 2006). Molecular dynamics simulations show the preferential alignment of water molecules near interface ions (Bhatt et al. 2004) and of water and CO₂ molecules at the interface (da Rocha et al. 2001, Kuznetsova & Kvamme 2002, Kvamme et al. 2007). These molecular-scale phenomena cause the emergence of a contractile membrane along the interface and a measureable surface tension $T_s$ [mN/m]. This situation applies to L-G, L₁-L₂ and L-I interfaces.

![Figure 2](image2.png)

**Figure 2. Effect of gas pressure on the development of surface tension along a gas-liquid interface.** (a) Low gas pressure. (b) High gas pressure.

The effect of dissolved species. Foreign species modify the electrical field within the liquid and alter the interfacial tension. Solutes may either be repelled by the interface or attracted to it (van Oss et al. 2002). Variations in interfacial tension $T_s$ [mN/m] with solute concentration $c$ [mol/L] are anticipated in terms of surface excess of solute $\Gamma$ [mol/m²] (Butt et al. 2006, Pegram & Record 2007, Tuckermann 2007),

\[ \frac{\partial T_s}{\partial c} \bigg|_T = -\frac{RT}{\gamma c} \Gamma \]  

where $\gamma$ [dimensionless] is the solute activity coefficient and $T$ [K] is temperature. Ions are depleted at the interface $\Gamma < 0$ in inorganic solutions, but there is enrichment of organic species $\Gamma > 0$ at the interface when organic compounds are present. In the case of water-CO₂, there is high concentration of dissolved CO₂ near the interface, $\Gamma > 0$, causing a drop in interfacial tension (Chun & Wilkinson 1995, Massoudi & King 1974, Sutjiadi-Sia et al. 2008).

The effect of temperature. The balance between thermal activity and molecular forces determines the evolution of surface tension with temperature. In general, surface tension decreases with increasing temperature.

The effect of pressure: fluid density. Consider a liquid-gas LG interface. The proximity to and the number of near-neighbor charges depends on gas density. Hence, higher interaction and lower interfa-
ter-CO₂, water-CH₄, water-oil, water-ice, and water-hydrate. Data for water-CO₂ in Figure 3 shows the pronounced sensitivity of interfacial tension with pressure and the relatively constant Ts values when pressure exceeds the CO₂ L-V boundary.

**Critical point.** A distinct separation between a liquid and its vapor ceases to exist and surface tension vanishes at P-T conditions above the critical point (Bauer & Patel 2009). The critical point for water is at 22MPa and 647K, and for CO₂ at and 7.4MPa and 304K. Supercritical fluid conditions develop above this pressure and temperature.

**Values.** Figure 3 and Table 2 show interfacial tension data for mixed fluid conditions relevant to energy geotechnology, including water-vapor, water-CO₂, water-CH₄, water-oil, water-ice, and water-hydrate.

[Figure 3. Interfacial tension between water and CO₂. Lines indicate values reported in the literature for deionized water at ~298K. Collected data and measured values reported in Espinoza & Santamarina (2010).]

![Figure 3. Interfacial tension between water and CO₂. Lines indicate values reported in the literature for deionized water at ~298K. Collected data and measured values reported in Espinoza & Santamarina (2010).](image)

<table>
<thead>
<tr>
<th>Pressure [MPa]</th>
<th>Interfacial tension [mNm]</th>
<th>Contact angle [°]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>10</td>
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<td>15</td>
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<td>15</td>
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<tr>
<td>20</td>
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<td>20</td>
</tr>
</tbody>
</table>

**Table 2. Interfacial tension and contact angle.**

![Table 2. Interfacial tension and contact angle.](image)

2.2 **Contact angle**

The contact angle formed by two adjacent fluids resting on a mineral substrate reflects the mutual interactions between the three neighboring phases.

**Contact angle and interfacial tension.** Fluid-fluid and fluid-solid interfaces attempt to shrink because of the corresponding interfacial tensions. The contact angle θ reflects the equilibrium between these forces. In particular, if a liquid is surrounded by its own vapor, the relevant surface tensions are the vapor-solid Ty-vs, liquid-solid Tls, and liquid-vapor TLv tensions.

The contact angle is computed from the sum of forces parallel to the solid surface (Fig. 4; Adamson & Gast 1997)

\[
\cos \theta = \frac{T_{YS} - T_{LS}}{T_{LV}}
\]

Young’s Equation (2)

The crystal structure of minerals explains the different affinity for fluids. For example, when crystallization takes place in an aqueous environment, groups with water affinity develop on the surface, and the contact angle that water forms on the crystal is lower than the angle observed in the same mineral crystallized in air (Shaw 1992).

**Oil-wet and water-wet mineral surfaces.** The assessment of wettability from contact angle measurements is depicted in Figure 4. By convention, the contact angle is measured with respect to water. Usually, mineral surfaces in soils and rocks are hydrophilic and water-wet conditions prevail. The oil-wet condition may result when a water-repellent agent (for example, silicone) is absorbed onto the mineral surface. Chemisorption of organic matter on the mineral surfaces may also create a hydrophobic surface. However, since mineral surfaces are hydrophilic in nature, the oil-wet condition can be considered a temporary condition. Thus, given enough time, organic fluids tend to be displaced by water. In short-time processes, wettability has a significant effect on multiphase flow, such as during enhanced oil recovery (Morrow 1990).

**Hysteresis.** The effects of temperature, pressure, chemical composition, and pH on interfacial tension and wetting of solid surfaces can be investigated by means of contact angle measurements (see techniques in Shaw 1992, Dullien 1992, Kwok & Neumann 1999). The apparent simplicity in the determination of contact angle is misleading. In particular, there is hysteresis in the contact angle θ with respect to the direction of fluid motion. There are two possible causes for contact angle hysteresis (Adamson & Gast 1997, Extrand 1998). First, there is the effect of surface roughness, whereby the macroscopically observed angle differs from the contact angle at the
scale of asperities, which is the scale relevant to Young’s equation. Second, the contact angle is affected by surface heterogeneity due to the alteration of surface properties by fluids and the presence of surfactants or surface-active agents.

**Pressure effects.** Changes in interfacial tensions $T_{VS}$, $T_{LS}$, and $T_{LV}$ (in particular) with pressure will alter the contact angle, particularly in liquid-gas-mineral systems such as water-CO$_2$-substrate and water-CH$_4$-substrate systems. For example, an increase in pressure from $P=0.1$MPa to ~8MPa causes an increase in contact angle of $\Delta \theta \approx 45^\circ$ (water on hydrophobized glass pressurized by CO$_2$), of $\Delta \theta \approx 50^\circ$ (Water droplet on Teflon-PTFE pressurized by CO$_2$), of $\Delta \theta \approx 60^\circ$ (water on coal pressurized by CO$_2$) (Dickson et al. 2006, Siemons et al. 2006, Chiquet et al. 2007, Chi et al. 1988). Additional data are shown in Figure 5 for water-CO$_2$-mineral systems. Note that contact angle decreases on hydrophilic surfaces but increases on hydrophobic surfaces when $T_{LV}$ decreases.

**Values.** Table 2 and Figure 5 provide contact angle data for mixed fluid conditions relevant to energy geotechnology, including water-vapor, water-CO$_2$, water-CH$_4$, water-oil, water-ice, and water-hydrate.

$$\Delta u = T_{LV} \left( \frac{1}{R_1} + \frac{1}{R_2} \right)$$  \hspace{1cm} \text{Young-Laplace} (3)
where $R_1$ and $R_2$ are the two principal radii of curvature. The same formulation permits computing the pressure difference between gas-water $u_{G-W}$, gas-oil $u_{G-O}$, ice-water $u_{I-W}$, hydrate-water $u_{H-W}$, and oil-water, $u_{O-W}$. Equation 3 anticipates that the pressure difference $\Delta u$ can be large when small pores are involved. Implications on the development of discontinuities are explored later in this manuscript in the context of fractures and lenses. In a cylindrical pore, the pressure difference between the wetting and the non-wetting fluids is (Fig. 7b)

$$\Delta u = \frac{2T_{LV}}{R} \cos \theta$$

The vapor pressure is affected by the curvature of the liquid-vapor interface. Relative humidity and capillary pressure relate as (Defay & Prigogine 1966):

$$\Delta u = \frac{\rho}{M} RT \ln \left( \frac{1}{h_r} \right)$$

where $T$ is absolute temperature, $R$ is the universal gas constant $R=8.31 \text{ N-m/mol-K}$, $M$ is the molecular weight of the liquid, and $\rho$ is the liquid density. Equilibrium in water-ice and water-hydrate systems is affected by temperature $T$ (Clennell et al 1999, Coussy 2005, Coussy & Monteiro 2007, Gens 2010):

$$\Delta u = \Sigma_m (T_m - T)$$

where $T_m$ is the melting temperature and $\Sigma_m$ is the melting entropy (for ice: $T_m=273.15K$ and $\Sigma_m=1.2 \text{ MPa/K}$). We can combine these equations for the case of cylindrical tubes to obtain the size of pores invaded by the non-wetting phase at given relative humidity or temperature conditions:

$$R = \frac{2T_{LV}}{\Sigma_m(T_m - T)} \cos \theta$$

for hydrate-water $u_{H-W}$, ice-water $u_{I-W}$, and oil-water, $u_{O-W}$.

Equations 7 & 8 relate invaded pore size to either relative humidity or temperature, while Equations 3 & 4 relate pore size to capillary pressure. These equations together with pore structure (connectivity and spatial variability) define the characteristic curves $\Delta u$-$S$ for the following systems under consideration: gas-water $u_{G-W}$, gas-oil $u_{G-O}$, ice-water $u_{I-W}$, hydrate-water $u_{H-W}$, and oil-water, $u_{O-W}$.

2.4 Solubility – Saturation

Phase A may dissolve or come out of solution from a neighboring phase B depending on pressure and temperature. During this process, interfaces may appear or disappear. When solubility limits for A-in-B and B-in-A are reached, an equal number of molecules from each species travel from one phase to the other.

$P-T$ dependent solubility. The $P-T$ dependent concentration of a certain specie in another phase $M_{P,T}$ [mol/m$^3$] can be approximated using a linear function of pressure

$$M_{P,T} = P_{\text{ applied}} k_H^0 \exp \left[ - \frac{\Delta H}{R} \left( \frac{1}{T} - \frac{1}{T_{298.15K}} \right) \right]$$

The enthalpy of the solution is $\Delta H = -14130 \text{ [J/mol]}$ for CH$_4$ in water, and $\Delta H = -19940 \text{ [J/mol]}$ for CO$_2$ in water (Wilhelm et al. 1977). Typical values of Henry’s constant for different gas species are listed in Table 3. Hence, the solubility of gas increases with increasing pressure and decreasing temperature, indicating an increased preference for dissolved gas rather than a separate phase at high pressure.

<table>
<thead>
<tr>
<th>Gas</th>
<th>Henry’s constant $k_H^0$ [M/atm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carbon dioxide CO$_2$</td>
<td>$3.4 \times 10^{-2}$</td>
</tr>
<tr>
<td>Methane CH$_4$</td>
<td>$1.4 \times 10^{-3}$</td>
</tr>
<tr>
<td>Hydrogen H$_2$</td>
<td>$7.8 \times 10^{-4}$</td>
</tr>
<tr>
<td>Nitrogen N$_2$</td>
<td>$6.5 \times 10^{-4}$</td>
</tr>
<tr>
<td>Oxygen O$_2$</td>
<td>$1.3 \times 10^{-3}$</td>
</tr>
<tr>
<td>Sulfur dioxide SO$_2$</td>
<td>1.2</td>
</tr>
</tbody>
</table>

Competing solutes. Solubility is affected by the emergence of new phases or the presence of competing solutes. For example, the presence of hydrates facilitates further hydrate formation and the equilibrium concentration of gas in water decreases in the presence of hydrate (Waite et al. 2009). Salt is a competing solute for dissolved gas and lowers the solubility of gas in water (Davie et al. 2004, Sun & Duan 2007, Tishchenko et al. 2005, Zatsepina & Buffett 1998).

Size-dependent solubility. Smaller water droplets in vapor, hydrate crystals in water, or gas bubbles in water have higher pressure, i.e., chemical potentials,
and require higher concentration (water or methane) in the surrounding fluid to balance that chemical potential (Henry et al. 1999, Kwon et al. 2008). This process is significant in small pores (e.g., smaller than ~40 nm for hydrate in seawater (Sun & Duan 2007)).

**Supersaturation.** Solutions are often supersaturated at prevailing pressure and temperature conditions. The degree of supersaturation $\delta$ is defined as:

$$
\delta = \frac{c_{\text{actual}}}{c_{\text{equil}}} - 1 \quad (10)
$$

Experimentally determined supersaturation values for different gases in water are listed in Table 4. The degree of supersaturation decreases if the solute nucleates as a separate phase and in the presence of certain impurities that facilitate (rather than hinder) nucleation.

Table 4. Measured values of supersaturation needed to cause bubble nucleation in aqueous solutions. Compiled from Lubetkin (2003).

<table>
<thead>
<tr>
<th>Gas</th>
<th>Measured supersaturation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carbon dioxide $\text{CO}_2$</td>
<td>4.62–20</td>
</tr>
<tr>
<td>Methane      $\text{CH}_4$</td>
<td>80</td>
</tr>
<tr>
<td>Hydrogen $\text{H}_2$</td>
<td>80–90</td>
</tr>
<tr>
<td>Nitrogen $\text{N}_2$</td>
<td>19–140</td>
</tr>
</tbody>
</table>

2.5 **Nucleation**

Gas bubble formation, salt precipitation and hydrate nucleation phenomena are end-conditions to solubility. Critical nuclei size and heterogeneous nucleation play a critical role in the formation of a separate phase.

**Heterogeneous nucleation.** Supersaturation thresholds for homogeneous nucleation in the bulk liquid are a function of molecular interactions between the liquid and the dissolved gas. The presence of impurities and surface imperfections may facilitate nucleation and lower the degree of supersaturation. For example, the presence of mineral surfaces tends to favor heterogeneous bubble nucleation at substantially lower supersaturations. Nucleation centers in sediments include microcavities, irregularities and impurities at mineral surfaces (Dominguez et al. 2000, Blander 1979, Gerth & Hemmingsen 1980, Pease & Blinks 1947).

The heterogeneous nucleation of hydrate and ice on mineral surfaces is prompted by reduced thermal activity and spatial distribution of water molecules in the vicinity of mineral surfaces. However, ice or hydrate cages do necessarily form against the mineral surface as hydrogen bonding is needed to form the crystals. We can observe that: (1) the boundary layer may extend to few monolayers; (2) water molecules are not in a disordered liquid form, (3) water molecules have limited mobility, (4) diffusion through this layer is necessarily lower than in the bulk fluid, and (5) the hydrate (or ice) and mineral interface may sustain a tensile strength.

**Critical nuclei size.** Molecules continuously jump from one phase to the other, e.g., gas molecules jump from the bubble into the liquid and vice versa (numerical simulations in Walsh et al. 2009). The effect of size on solubility implies that molecules could jump out of small “embryos” faster than they could jump back into them. There is a critical nuclei size $d^*$ for which nuclei are in equilibrium with the solution so that nuclei smaller than $d^*$ will tend to dissolve (Nyvlt et al. 1985, Finkelstein & Tamir 1985, La Mer 1952, Ward et al. 1970, Lubetkin 2003). Typically, stable nuclei are several nanometers in size and $d^*$ decreases with supersaturation.

**Ostwald ripening.** The higher saturation predicted around smaller gas bubbles or crystalline nuclei promote diffusion from small nuclei towards large ones. Therefore a large bubble or crystal will tend to grow at the expense of smaller neighboring nuclei. This process is called Ostwald ripening. Ripening is a diffusion-controlled aging process and is important in fast crystallization processes that produce small crystal sizes. Ostwald ripening will alter the crystal size distribution with time (Myerson 2002).

**Gas nucleation.** Gas molecules occupy cavities between water molecules or between organic molecules in oil, until the fluid reaches the supersaturation threshold that prompts bubble nucleation (Ronen et al. 1989). Spontaneous bubble nucleation can result from: (1) depressurization of a pure liquid below the vapor pressure, (2) temperature increase for a pure liquid until the vapor becomes more stable than the pure liquid, (3) by gas coming out of solution from a supersaturated liquid (e.g., due to depressurization as per Henry’s law; biogenic gas at constant pressure), or (4) by hydrate dissociation when $P-T$ conditions migrate outside the stability field (Hemmingens 1975 & 1977, Lubetkin 2003, Rebata-Landa & Santamarina 2010, Santamarina & Jang 2009).

**Ice and hydrate nucleation.** Ice forms at low temperature when the thermal activity of water molecules decreases to allow a solid structure. The presence of molecules such as $\text{CH}_4$ or $\text{CO}_2$ helps the nucleation of water cages around these gas molecules. The resulting crystalline structure is known as gas hydrate. Typical structures are shown in Figure 8a. The $P-T$ phase boundaries for ice, $\text{CO}_2$ hydrate and $\text{CH}_4$ hydrate are shown in Figure 8b and c.
Ice and hydrates share similar properties. There are, however, two important differences. First, the phase boundary for ice is at almost constant temperature independently of pressure, while the phase boundary for \( \text{CH}_4 \) and \( \text{CO}_2 \) hydrates is pressure and temperature dependent. Differences in \( P-T \) phase boundaries define the location of permafrost (cold regions) versus the location of hydrates (offshore & below deep permafrost, i.e., high pressure & low temperature).

Second, water expands during ice formation; conversely, ice contracts as it melts (~8.3% volume contraction). The case of hydrates is more complex: the initial volume of water \( V_w \) increases to \( V_{\text{hyd}}=1.26V_w \) when methane hydrates form; upon dissociation, hydrates form water and gas phases and the initial volume of hydrate \( V_{\text{hyd}} \) expands to occupy a volume \( \beta V_{\text{hyd}} \) where \( \beta \) is \( P-T \) dependent: for \( P=10\text{MPa} \) and \( T=288.15\text{K} \), \( \beta=2.26 \), the water occupies \( V_w=0.79V_{\text{hyd}} \) and the rest is the volume of gas \( V_g=1.47V_{\text{hyd}} \). These pronounced volume expansion upon dissociation and the formation of separate gas and liquid phases anticipate significant implications of dissociation.

Gases and non-wetting fluids invade pores by displacing water, starting at large pores and gradually advancing into smaller pores, according to the interconnected pore structure. Ice and hydrate also invade the pore space along the larger-size interconnected porosity, but do so by crystallizing in pores.

3 PORE AND PARTICLE-SCALE ANALYSES

Interfacial tension, contact angle, solution / dissolution and formation / dissociation processes occur at the pore scale and affect the sediment response through particle-scale interactions. Related concepts and phenomena are analyzed next.

3.1 Pores and pore throats

Size – Statistics. The evolution of mixed-fluid phenomena in porous/granular media is intimately related to pore size distribution and spatial correlation.

The mean pore size \( \mu(d_p) \) can be analytically estimated as a function of the void ratio \( e \), the specific surface \( S_s \) [m\(^2\)/g] and the mineral density \( \rho \) [g/cm\(^3\)]

\[
\mu(d_p) = k \frac{e}{S_s \rho}
\] (11)

where the shape factor is \( k \approx 5 \) for spherical particles, \( k=2 \) for dispersed clays and \( k=4 \) for edge-to-face configuration. The void ratio depends on the effective stress \( \sigma' \) as

\[
e = \epsilon_{1\text{kPa}} - C_c \log\left(\frac{\sigma'}{1\text{kPa}}\right)
\] (12)

where \( \epsilon_{1\text{kPa}} \) is the void ratio at \( \sigma'=1\text{kPa} \) and \( C_c \) the compressibility coefficient of the sediment.

We have compiled published pore size distribution data for clays, silts, and sands. Results show that pore size distribution statistics (mean and standard deviation) increase with particle size. When data are fitted with a log normal distribution, the standard deviation \( \sigma \) of \( \ln(d_p/\mu\text{m}) \) is bound between (Phadnis & Santamarina 2010)

\[
\sigma \left[ \ln\left(\frac{d_p}{\mu\text{m}}\right) \right] = 0.4 \pm 0.2
\] (13)

Finally, we note that pores are connected through pore throats. The size of pore throats is related to the size of the two connected pores; assuming a simple cubic configuration,
Maximum capillary pressure. The pressure difference between the non-wetting and wetting phases $\Delta u_{\text{max}}$ is limited by the largest pore throat along the boundary between wetting and non-wetting fluids

$$\Delta u_{\text{max}} = u_{\text{nw}} - u_{\text{w}} \leq \frac{4T_s}{\max(d_{th})}$$

This pressure difference can develop in gas, oil, ice or hydrate phases in water-saturated sediments.

Upscaling – network models. Pore-scale phenomena upscale through the interconnected pore space. The interconnected porosity can be captured using tube-only, pore-only, and pore + tube network models. Tube-network models are used to compute flow conditions, assuming Poiseuille flow within each tube

$$q = \frac{8\mu \Delta P}{\pi R_{\text{tube}}^4}$$

where the flow rate through a tube $q$ [m$^3$/s] is a function of fluid viscosity $\mu$ [N·s/m$^2$], tube radius $R_{\text{tube}}$ [m], tube length $\Delta L$ [m], and pressure difference between end nodes $\Delta P$ [N/m$^2$]. These models allow us to study fluid flow even under mixed fluid flow conditions. On the other hand, pore-networks are better suited for the study of unsaturation; in this case, each pore is connected to its neighboring pores through throats that satisfy geometric constraints relative to the log-normal distributed pores. A comparative summary is presented in Figure 9.

3.2 Particle size – Particle level forces

Sediments are particulate materials. Particle displacement, hence the macroscale sediment mechanical response, is determined by the forces particles experience. These include forces generated at the boundary and transmitted through the skeleton (i.e., due to effective stress), forces that result from the particle volume (weight and buoyancy), forces that develop along the particle surface (hydrodynamic and capillary) and contact level forces (electrical and cementation-reactive).

Asymptotic, order-of-magnitude estimates of the forces most relevant to this manuscript are summarized in Table 5. The normal contact force $N$ related to the applied effective stress $\sigma'$ scales with the square of the particle diameter $N=\delta^2 \sigma'$ (assuming a simple cubic packing of equal size spheres). Weight and buoyancy combine to determine the submerged weight of a particle of unit weight $\gamma$ submerged in a fluid of unit weight $\gamma$, $S=\pi(\gamma_d-\gamma)d^3/6$.

<table>
<thead>
<tr>
<th>Force Type</th>
<th>Formula</th>
</tr>
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<tbody>
<tr>
<td>Skeletal</td>
<td>$N=\delta^2 \sigma'$</td>
</tr>
<tr>
<td>Weight</td>
<td>$W=\pi R_d \gamma_d d^3/6$</td>
</tr>
<tr>
<td>Buoyant</td>
<td>$U=\pi \Delta \gamma d^3/6$</td>
</tr>
<tr>
<td>Hydrodynamic</td>
<td>$F_d=5\pi \mu \nu d$</td>
</tr>
<tr>
<td>Capillary</td>
<td>$F_c=2\pi T_s d$</td>
</tr>
<tr>
<td>Electrical Attraction</td>
<td>$A_{\text{hd}}=4\pi d^2/24t^2$</td>
</tr>
<tr>
<td>Electrical Repulsion</td>
<td>$R_{\text{hd}}=0.0024 c_{\text{hd}}^3 \text{exp}(-10^4 c_{\text{hd}}^{1/2}) \text{d}$</td>
</tr>
<tr>
<td>Cementation</td>
<td>$T_{\text{hd}}=\pi \sigma_{\text{hd}}$</td>
</tr>
</tbody>
</table>

Re.: Cho et al (2001), Santamarina (2001); Gili & Alonso 2002

The capillary force a particle experiences when the sediment is subjected to mixed fluid conditions can be estimated using sketches shown in Figure 10 (Note: this analysis applies to vapor-water, gas-oil, oil-water, ice-water and hydrate-water mixtures). There are two force components. One is exerted by the interfacial membrane in the direction of the wetting fluid, as the interfacial membrane clings to the mineral around the grain surface. The second component is caused by the pressure difference between the wetting and the non-wetting phase against the cross section of the grain. Then, the capillary force acting on the grain and transmitted to the sediment skeleton is (compressive towards the side of the wetting fluid)

$$F_{\text{cap}} = \pi d T_s \cos \theta + \frac{\pi d^2}{4} (u_{\text{nw}} - u_{\text{w}})$$

$$\leq \pi d T_s \cos \theta + \frac{\pi d^2}{\max(d_{th})}$$

$$\leq 2\pi d T_s \text{ (for uniform grain size)}$$

Figure 9. Networks of tube and of pores used for mixed fluid studies (details in Jang and Santamarina 2010a&b - See also Blunt 2001 for a comprehensive review of network models).
Upscaling – grain models. Particle-scale forces determine the equilibrium of each particle and its eventual displacement. Sediment-scale implications can be explored using particle models, which are based on Newtonian mechanics. The resultant force $F$ acting on a given particle of mass $m$ causes it to accelerate with $a=F/m$. After a time interval $\Delta t$, the particle is at a new location $\Delta x=v_0\Delta t+at^2/2$ and experiences a new set of forces.

4 MACRO-SCALE PHENOMENA: MIXED FLUID FLOW

Pore-scale and particle-scale conditions identified in previous sections are used herein to explore important phenomena that take place in sediments in the context of energy geotechnology.

4.1 Fluid invasion - Mixed fluid conditions

Fluid invasion under mixed fluid conditions is controlled by capillary $F_c$, viscous drag $F_d$ and weight/bouyancy $W$ or $F_b$ forces. These forces can be combined into dimensionless numbers (Pennel et al. 1996, Lenormand et al. 1988 – Note: the defending and invading fluid viscosities are $\mu_{\text{def}}$ and $\mu_{\text{inv}}$):

$$N_C = \frac{F_d}{F_c} = \frac{3\pi d\mu_{\text{inv}} v}{\pi d T_s \cos \theta} = \frac{3\mu_{\text{inv}} v}{T_s \cos \theta}$$

(18)

$$N_M = \frac{\mu_{\text{inv}}}{\mu_{\text{def}}}$$

(19)

$$N_G = \frac{F_d}{W} = \frac{3\pi d \mu_{\text{inv}} v}{\pi G_s \gamma_s d^3 / 6} = \frac{18\mu_{\text{inv}} v}{G_s \gamma_s d^2}$$

(20)

$$N_b = \frac{F_b}{F_c} = \frac{\Delta \gamma \pi d^2/6}{\pi d T_s \cos \theta} = \frac{\Delta \gamma d^2}{6 T_s \cos \theta}$$

(21)

Invasion conditions and emergent phenomena can be identified in the dimensionless space of these $\pi$-ratios. Figure 11 shows a 2D-slice of this space.

4.2 Breakthrough pressure

The long-term storage of CO$_2$ is a quasi-static condition controlled by capillary forces at pore throats. Similar conditions apply to gas and oil storage. The distribution of pore size discussed in Section 3.1 is invoked in this analysis. For a given pore structure, the breakthrough pressure $p_{c^*}$ determined by the pressure-dependent interfacial tension $T_s$ and contact angle $\theta$ is given by

$$p_{c^*} = \frac{4T_s \cos \theta}{d_p^*}$$

(22)

where $d_p^*$ is the minimum pore diameter along a percolating path across the seal layer. The pore size

Figure 10. Forces on particles under mixed fluid flow condition (Shin & Santamarina 2010b).

Figure 11. Space for viscous fingering, capillary fingering, and stable displacement in terms of dimensionless ratios $N_M$ and $N_C$ – refer to text (modified from Lenormand et al. 1988).
$d_p^*$ can be related to the mean by a factor $\alpha$ of the standard deviation. For a log-normally distributed pore size,

$$\ln \left( \frac{d_p^*}{nm} \right) = \mu \left( \ln \frac{d_p}{nm} \right) + \alpha \sigma \left( \ln \left( \frac{R_p}{\mu m} \right) \right)$$  \hspace{1cm} (23)

The analysis of gas breakthrough data in Hildenbrand et al. (2002), in Hildenbrand et al. (2004) and in Horsey et al. (1999) leads us to conclude that percolating paths are made of pores larger than the mean, and that the second term in Equation 23 is relatively independent of effective stress (Espinoza & Santamarina 2010). We note that this analysis does not take into consideration poro-mechanical effects (pore size is affected by the percolating immiscible fluid and the ensuing capillary forces – Section 4.6) and reactive fluid transport (which also affects the pore size along the percolating path – Section 4.9)

4.3 Gas invasion -vs.- internal gas nucleation

Expressions for degree of saturation and permeability as a function of capillary pressure have been derived for typical drying soil conditions where water vapor is in a continuous phase with the atmosphere.

These $\Delta u$-$S$ expressions can be used to investigate similar conditions in energy geotechnology, such as the injection of liquid CO$_2$ into water-saturated sediments. However, gas comes out of solution and bubbles grow within the sediment in various energy-related applications, from oil recovery to methane production from hydrate bearing sediments (as well as frequent seepage conditions downstream of earth dams). These two cases are referred to “internal gas drive process” for the case of nucleation and gas liberation during depressurization, and “external gas drive process” or gas injection for the case of forced invasion (Yortsos & Parlar 1989, Poulsen et al. 2001, Nyre et al., 2008).

We use 3D network models to explore the effect of gas invasion versus nucleation on the evolution of the characteristic saturation curves and relative permeabilities (details in Jang & Santamarina 2010c). We assume slow invasion and nucleation so that the process is controlled by capillary forces – both viscous and gravity forces are disregarded (refer to Section 4.1). Gas invasion is enforced from a peripheral gas-water interface, while gas nucleation is initiated at randomly selected internal nodes; the algorithm and tube size distribution are identical in both invasion and nucleation simulations. Results are described in terms of the defending fluid retention and relative permeabilities.

**Characteristic curve in random media.** The characteristic curve captures the causal link between water saturation and capillary pressure (See also Wilkinson & Willemsen 1983, Lenormand & Zarcone 1984). Pore throat size distribution and spatial correlation govern the shape of the characteristic curve (Francisca & Arduino 2007). The lower bound characteristic curve is determined by sorting pores and gradually invading from the largest pore to the smallest one; conversely, the upper bound is obtained by invading the smallest pore first and blasting through the sediment as gradually larger pores are invaded. Real cases fall in between these two extremes. The characteristic curves for spatially-uncorrelated random-distributed pores are shown in Figure 12a (Simulation details are indicated in the figure caption). Results are almost identical for gas invasion and gas nucleation. Water has been displaced in most tubes when the capillary pressure corresponds to the mean pore size, which in this case is $\Delta u=0.14$MPa for $\mu(R_{nuc})=1$µm.

**Relative permeabilities.** Gas and water conductivity during gas invasion and nucleation are calculated at each saturation. Computed water conductivities are normalized by the water conductivity of the fully water saturated network (Figure 12b). Gas conductivities are normalized by the gas conductivity obtained when gas invasion process is completed. The normalized water conductivities are almost same for both gas invasion and nucleation.

![Figure 12. Characteristic curves and relative permeabilities - Gas invasion vs. gas nucleation. (a) Characteristic curves. Symbols: (○) gas invasion through 13$^2$ nodes on one side, (●) gas invasion into the network sorted by tube size; gas invasion through multiple nodes distributed inside network model: (□) 13$^2$, (●) 2×13$^2$, (◇) 3×13$^2$ nodes. (b) Relative conductivity of water $k_{rw}$ and gas $k_{rg}$. Results obtained using a three dimensional tube-network model. Details: 13×13×13 nodes, 5460 tubes, coordination number $cn=6$, log-normal distribution of tube radius $R$, the mean tube size $\mu(R)=1$µm, and the standard deviation in tube radius $\sigma(\ln(R/\mu m))=0.4$. Parameters for Young’ equation $P_r=2T_{ll}/\cos\theta/R$: $T_{ll}=72$ mN/m, $\theta=0^\circ$.](image-url)
However, gas conductivity during nucleation is much lower than during gas invasion. Similar conclusions were reached by Poulsen et al. (2001) using different model characteristics.

**Other implications.** There are important poro-granular mechanical distinctions between gas invasion and nucleation conditions. First, we observe that isolated bubbles can exist at different gas. Second, a capillary front evolving at the sediment boundary (e.g., during desiccating) tends to cause sediment compaction, while internal nucleation promotes expansion. These two cases are discussed later in this section, in the context of desiccation cracks and bubble migration.

### 4.4 Residual saturation – recovery efficiency

Oil recovery efficiency is controlled by pore connectivity and water invasion. The recovery of gas from hydrate bearing sediments depends on volume expansion upon dissociation.

**Oil recovery.** The displacement efficiency $E_O$ during oil recovery can be defined in terms of the initial oil saturation $S_{oi}$ and the residual oil saturation $S_{or}$ as

$$E_O = \frac{S_{oi} - S_{or}}{S_{oi}}$$

Residual oil saturation after water flooding ranges from ~10% to ~40%, depending on the sediment heterogeneity and flooding conditions (Morrow 1990).

**Gas hydrates.** Internal gas nucleation and gas expansion creates conditions that are different to oil extraction. We use pore-network simulations to investigate gas recovery efficiency and residual gas saturation. An analytical result can also be obtained for gas recovery efficiency $E_G$ using macro-scale analyses (details in Jang & Santamarina 2010b)

$$E_G = \frac{V_{g}^{\text{res}}}{V_{g}^{\text{dis}}} = \frac{V_{g}^{\text{fin}} - V_{g}^{\text{res}}}{V_{g}^{\text{fin}}} = \frac{\beta V_{h}^{\text{ini}} - 0.79 V_{h}^{\text{ini}} - V_{g}^{\text{res}}}{\beta V_{h}^{\text{ini}} - 0.79 V_{h}^{\text{ini}}}$$

$$= \frac{\beta - 0.79 - S_{g}^{\text{res}}/S_{hyd}}{\beta - 0.79} \quad (0.21 S_h \leq S_{g}^{\text{res}} \leq 1) \quad (25)$$

where the $V_{h}^{\text{ini}}$ is initial hydrate volume and $V_{g}^{\text{res}}$ is residual gas saturation per unit volume of sediment, $S_{hyd}$ is initial hydrate saturation, $\beta$ represents the expansion of dissociated gas and water from initial volume of hydrate. Numerical and analytical results presented in Figure 13.

Results show that gas recovery efficiency and residual gas saturation during hydrate dissociation are functions of initial hydrate saturation, mean pore size, and the variability in pore size distribution.

### 5 MACRO-SCALE PHENOMENA: MECHANICAL EFFECTS

The ratio $F_{cl}/N=2\pi T_s/(\sigma' d)$ –refer to Table 5- defines two extreme regimes: mixed fluid conditions do not affect the mechanical behavior of the sediment if $F_{cl}/N<<1$, conversely capillary phenomena will have a profound effect on the sediment mechanical response when $F_{cl}/N>>1$ (Fig. 14). Clearly, this analysis can be extended to include other particle level forces, such as cementation and electrical attraction. Salient phenomena are discussed next.
Figure 14. Capillary force vs. skeletal force. Localizations develop at high $F_c/N$ ratio.

5.1 Seepage and gas nucleation: bottom blow up

Bubble nucleation during depressurization and associated changes in hydraulic conductivity may combine to create unwanted failure conditions (we acknowledge studies by Japanese researchers in the early 1990’s). Consider upward flowing liquid through a soil plug of length $H$. The liquid pressure at the lower inlet is $u_i$, and $u_o$ at the upper outlet. The liquid pressure when gas comes out of solution is the bubble point $u_b$ (Note: the liquid is not gas saturated at pressure $u_i$).

Let’s assume that equilibrium has been reached so that the continuity of water flow is maintained throughout the saturated and unsaturated soil layers $q_w^{(s)}=q_w^{(u)}$, and that the hydraulic conductivities are related as $k^{(u)}=\alpha k^{(s)}$ (Fig. 15).

Then, the elevation $L$ where gas comes out of solution can be computed from the following quadratic equation in $L$ (the explicit solution for $L$ is not shown here)

$$\frac{u_i-u_b}{\gamma} = \frac{u_b+L}{\gamma} - \frac{u_o+H}{\gamma}$$

$$L = \frac{\alpha \gamma \gamma}{\alpha \gamma + \gamma}$$

(26)

5.2 Development of discontinuities – granular mechanics effects

Desiccation cracks - Immiscible fluid cracks. The fundamental particle-level mechanism for desiccation crack initiation and growth is centered on the air-water interface membrane. Initially the air-water interface membrane resists invading pores, capillary suction increases, the effective stress increases, the soil consolidates, and the skeletal stiffness increases. Eventually, the air-water interface membrane invades the largest pores and causes particle displacement away from the invasion point. The void ratio increases at the tip and facilitates further membrane invasion and crack growth (Fig. 10 & 16). The same mechanism is responsible for fluid-driven fractures when other immiscible fluids are involved, such as gas-oil, gas-water, and oil-water systems.

Ice and hydrate lenses. The formation of segregated ice and hydrate lenses responds to similar force-balance conditions and the granular-mechanics effects at the tip: lens growth causes an increase in porosity at the tip, fluid migrate toward the tip and further nucleation is favored.

The growth of ice lenses is temperature controlled, and they typically develop parallel to the soil surface in frozen ground due to the advancing cold front, that is normal to the $\sigma_3$-direction (see Gens 2010). However, hydrate growth is gas-limited in most cases; in other words, sediments are already within the $P-T$ stability field and hydrate grows as methane becomes available. In this case, hydrate lens growth is facilitated normal to the minor principal stress $\sigma_3$ (evidence in Yun 2005).
Localization regime. As the particle size decreases, the particle weight and the skeletal force decrease faster than either the capillary force and the force exerted by the ice or hydrate lens. Hence, fine-grained soils are more susceptible to fluid-driven fractures and lens formation (Fig. 17 – see also Jain & Juanes 2009).

5.3 Buoyancy and bubble migration

Bubbles larger than the critical size of nuclei \( d > d^* \) can coalesce to form even larger bubbles. These large bubbles become trapped at pore throats defining Laplacian capillary surfaces. As the bubble size increases, so does the buoyancy drive and its tendency to migrate upwards (experimental evidence in Boudreau et al. 2009).

Buoyancy drive applies to other light and immiscible fluids. Consider a gas bubble that extends from depth \( z \) to depth \( z + L \), composed of a gas of unit weight much lower than the unit weight of water \( \gamma_{\text{g}} < < \gamma_{\text{w}} \). If the capillary pressure at the bottom of the bubble is \( \Delta u_{\text{bottom}} \), the capillary pressure at the top is equal to \( \Delta u_{\text{top}} = \Delta u_{\text{bottom}} + L \gamma_{\text{w}} \) (Fig. 18).

5.4 Monitoring: bulk stiffness

The monitoring of energy related processes is often conducted with \( P \)-wave propagation techniques. The stiffness of small bubbles has to be given special attention.

Bubbles much smaller than soil particles can fit within the pore space without distorting the soil structure. Thus, the presence of gas bubbles only changes the compressibility of the pore fluid (Wheeler 1988). Even relatively small size bubbles are sufficient to significantly change the pore fluid bulk stiffness (Sparks 1963, Santamarina et al. 2001). The pore fluid bulk stiffness \( \kappa_f \) depends on the degree of saturation \( S \), the bulk stiffness of water \( \kappa_w \approx 2.2 \text{GPa} \) and the bulk stiffness of gas bubbles \( \kappa_b \). The bulk stiffness of bubbles is intimately related to the gas pressure, which is governed by the surface tension and the bubble size \( d_b \). We can show that (Rebata-Landa & Santamarina 2010):

\[
\kappa_b = \frac{4 T_s}{3 d_b} \tag{28}
\]

Then, the fluid bulk stiffness of water becomes:

\[
\kappa_f = \frac{1}{S \frac{1}{\kappa_w} + (1 - S) \frac{3 d_b}{4 T_s}} \tag{29}
\]

Finally, the bulk stiffness of sediments in the presence of disseminated gas bubbles is:

\[
\kappa_{\text{soil}} = \kappa_{sk} + \frac{1}{n \left[ S \frac{1}{\kappa_w} + (1 - S) \frac{3 d_b}{4 T_s} \right]^2 + \frac{1-n}{\kappa_g}} \tag{30}
\]
5.5 Reactive fluid transport

Fluids react with the mineral walls and alter the sediment structure. Consider carbon dioxide: CO$_2$ dissolves in water to form carbonic acid H$_2$CO$_3$, which eventually ionizes into H$^+$ and CO$_3^{2-}$, lowering the pH of water. Minerals dissolve in acid water with dissolution rates that are proportional to the pH difference with respect to the buffer pH. The solubility of CO$_2$ increases with pressure (Stumm & Morgan 1996). Conversely, a reduction in pressure produces the nucleation of CO$_2$ gas and the precipitation of dissolved minerals. Therefore, both dissolution and re-prefication can take place in the sediment.

Mineral dissolution in the pore fluid satisfies similar concepts to those discussed in Section 2.4, albeit the low saturation concentration. Mineral dissolution by acidic water flow is most relevant to long-term geological storage of CO$_2$. Similar situations apply to other energy geotechnology systems including hydrate dissolution by unsaturated water flow. Three potential emergent phenomena are discussed next.

**Dissolution pipes (Hydro-chemical coupling).** A positive feedback condition develops between dissolution and preferential fluid flow paths. Eventually, high conductivity flow channels or “wormholes” may emerge (Hoefner & Fogler 1988; Fredd & Fogler 1998). Damköhler’s number $Da = \frac{\text{dissolution rate}}{\text{advective transport rate}}$, and Peclét’s number $Pe = \frac{\text{advective transport rate}}{\text{diffusion transport rate}}$ are used to explore the phenomenon: wormholes form at high $Pe > 1$ and $Da > 1$ numbers (Golfier et al. 2002).

**Change in capillary forces (mixed-fluid skeleton coupling).** Pore size enlargement during mineral dissolution or pore size reduction during precipitation alters capillary forces when immiscible fluids are involved.

**Change in internal skeletal stresses (chemo-mechanical coupling).** Mineral dissolution produces a pronounced drop in horizontal effective stress under zero lateral strain conditions. The state of stress decreases from a $k_r$-state and reaches the active particle flow $k_m$-state after ~5% mineral dissolution (Shin & Santamarina 2009). Furthermore, shear localization may take place during mineral dissolution in sediments that exhibit postpeak strain softening behavior (in agreement with Rudnicki & Rice 1975 - Details in Shin et al. 2008).

6 CONCLUSIONS

Problems related to energy geotechnology often involve mixed-fluid conditions. However, contrary to near-surface unsaturated soil mechanics, most energy geotechnology problems are at high fluid pressure and in deep sediments. This leads to phenomena seldom encountered in the more classical unsaturated soil literature. Processes reviewed in this manuscript are relevant to oil and gas, hydrate-bearing sediments and carbon geological storage. The most important observations follow.

- Surface tension is pressure-dependent. Contact angle is determined by the interfacial tension of neighboring phases, thus, it is pressure dependent as well.
- Capillary phenomena can be extended to include ice and hydrates. While the phase boundary for ice is almost constant with pressure, the stability field for hydrates is pressure and temperature dependent.
- Solubility is pressure and temperature dependent, and it is affected by the presence of other phase or solutes. Supersaturation facilitates nucleation. The higher saturation around smaller nuclei leads to diffusion-controlled ripening.
- Ice formation and melting is accompanied by volume change and affects the granular skeleton. Volume change is much more pronounced when gas hydrates are involved.
- The nucleation of a gas phase in pores reduces the sediment bulk stiffness, alters the hydraulic conductivity, and may lead to a zero effective stress condition, as observed in bottom blowup situations.
- Capillary pressure is a function of interfacial tension and curvature, and is related to relative humidity (water-vapor) or temperature (ice or hydrate). Hence, the size of pores invaded by the non-wetting phase can be related to relative humidity (vapor) or temperature (ice and hydrate).
- Pore size distribution and spatial connectivity govern fluid flow, percolation, break-through pressure, residual saturation and resource recovery efficiency. The percolating path for an immiscible fluid is made up of pores within the upper-side tail of the pore size distribution.
- The characteristic saturation curves $\Delta u-S$ show no significant difference for gas invasion and gas nucleation, and there is almost no difference for liquid relative permeability in these two cases. However, the gas relative permeability is lower for internal gas nucleation compared to gas invasion.
- Particle size distribution determines pore size and particle-level forces, which eventually define the sediment response. Forces are of gravitational, skeletal (related to effective stress), viscous, capillary, and electrical origin. These forces can be combined...
into dimensionless ratios to facilitate the identification of physical regimes and emergent phenomena.

- Pore size changes as immiscible fluids cause particle-level capillary forces and particle displacement. Immiscible fluid-driven fractures, desiccation cracks, and gas bubble migration reflect these particle-scale mechanisms.

- The invading fluid may react with minerals. Reactive fluid transport alters the sediment structure, induces localized flow, forms high conductivity flow channel, and changes skeletal stresses. Storativity and the performance of seal layers is limited by these emergent phenomena.

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NOTATION

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