Integrated Ground Behavior

Part A: Particle-level Phenomena & Macroscale Behavior (J. Carlos Santamarina)

Georgia Institute of Technology
forces
specific surface
relative size
sphericity
roundness
smoothness
conduction
diffusion
energy coupling
stiffness
threshold strain
strength
(scales)
rheology
diagenesis

soil

fluid

particles

mineral

size

shape

surface charge
double layers
capillarity
forces
specific surface
relative size
sphericity
roundness
smoothness
fine-grained soil fabric
coarse-grained soil packing
platy particles
Water

Ionic concentration

$pH = 7$

\[ H^+ \quad OH^- \]

\[ HCl \quad NaOH \]
Minerals

kaolin

\[ \text{OH}^- \quad \text{Al octa} \quad \text{Si tetra} \]

\[ \text{O}^= \]

calcite

montmorillonite

\[ \text{Si tetra} \quad \text{Al octa} \quad \text{Si tetra} \]

MDL / www.soils.wisc.edu/virtual_museum/index.html
Diffuse double layer

$\psi \propto e^{-x}$

Stern layer

Gouy diffuse layer

mineral surface

bulk electrolyte

pH ionic concentration & valence

$O^=\ OH^-$
Mixed fluid phase

*Interacting menisci:*

$\theta$ non-constant
Particle Forces

**Skeletal**
\[ N = \sigma v^2 d^2 \]

**Weight**
\[ W = (\pi G_s \gamma_w / 6) d^3 \]

**Buoyant**
\[ U = (\pi \gamma_w / 6) d^3 \]

**Hydrodyn.**
\[ F_{\text{drag}} = 3\pi \mu v d \]

**Capillary**
\[ F_{\text{cap}} = \pi T_s d \]

**El. attraction**
\[ \text{Att} = \frac{A_h}{24t^2} d \]

**El. repulsion**
\[ \text{Rep} = \frac{24\sqrt{\gamma_o}}{10000} e^{-10^8 t\sqrt{\gamma_o}} d \]

**Cementation**
\[ T = \pi \sigma_{\text{ten}} t d \]
Size and Surface - Surface Phenomena

![Graph showing specific surface area vs. particle size](image)

- **Specific Surface Area** [$\text{m}^2/\text{g}$]
  - $10^4 \text{m}^2/\text{g}$ (max for any mineral)
  - $10^2 \text{m}^2/\text{g}$
  - $1 \text{m}^2/\text{g}$
  - $10^{-2} \text{m}^2/\text{g}$
  - $10^{-4} \text{m}^2/\text{g}$

- **Particle Size** [$\text{nm}$, $\mu\text{m}$, mm]
  - montmorillonite
  - illite
  - kaolinite
  - Si flour
  - Ottawa 20-40

**formula:**

$$S_s = \frac{\sim 6}{L_{\text{min}} \rho}$$
Size and Surface - Surface Phenomena

Cum. Distribution

Surface  Weight

Density

Grain size
Fines migration and bridge formation

\[ \frac{D}{d} = 2.4 \]

\[ \frac{D}{d} = 6.5 \]

- MICA: clog → pass
- GLASS BEADS: clog → pass
- SAND: clog → pass

Pore size / \( d_{\text{fines}} \)
Particle Shape

- **sphericity** vs. ellipticity..platiness
- **roundness** vs. angularity
- **smoothness** vs. roughness

- Ottawa
- PCC
- Nevada
- formineferan
Particle Shape

<table>
<thead>
<tr>
<th>50 μm</th>
<th>chemically controlled</th>
<th>mechanically controlled</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><img src="image1" alt="kaolinite" /></td>
<td><img src="image2" alt="Margaret River" /></td>
</tr>
</tbody>
</table>

kaolinite

Margaret River
Fabric map

pH

Edge IEP, pH≈7.2

Face or Particle IEP, pH≈4

Silica dissolution - coagulation (pH>8)

deflocculated-dispersed

No global repulsion - vdw

deflocculated-dispersed

particle dissolution - liberation of Al\(^{3+}\) - coagulation (pH<2)

\[ c_0 = 0.1-0.15 \text{ mol/L} \]

Stern potential and \( R_{DL} \) decrease
dan der Waals attraction prevails

kaolinite
Clay minerals – Differences

different faces
different from edge
edge contribution
charge change
ζ / particle thickness
hiding edge DL
Coarse particles: relative size

D/d=1.37

D/d=5.3

random packing:

volume fraction of small particles

porosity, n

0.36

0.32

0.28

0.24

0

20

40

60

80

100%

(Guyon et al., 1987)

and...
Relative size and shape

(Krumbein and Sloss, 1963)

roundness = \sum \frac{r_i}{N} / r_{max}

max e_{min}

min e_{max}

minimum e_{min}

coefficient of uniformity, C_u

Yoed, 1973; see also Maeda, 2001
Platy particles: bridging

sand + mica

% mica

Lm/ds = 3
Lm/ds = 2
Lm/ds = 1
forces
specific surface
relative size
sphericity
roundness
smoothness

mineral
surface charge
double layers
capillarity

size
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soil
fine-grained soil fabric
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fluid

particles

microorganisms

conduction
diffusion
energy coupling

stiffness
threshold strain

strength
(scales)
rheology
diagenesis
Small-strain stiffness: Contact-controlled

\[ V_S = \alpha \left( \frac{\sigma'}{kP_a} \right)^{\beta} \]

where \( \beta = \frac{1}{6} \) for coarse grains and \( \beta < \frac{3}{4} \) for fine grains.

(Frocht, 1941)

(N. Skipper – UCL 2002)
Stiffness-Stress: contact behavior & fabric changes

\[ V_s = \alpha \left( \frac{\sigma'_x + \sigma'_y}{2 \text{kPa}} \right)^\beta \]

\[ \beta = 0.36 - \frac{\alpha}{700} \]
Stiffness: platy particles

Load [kPa] vs. [m/s]

$L_{m}/d_s = 3$

100% sand
1% mica
5% mica
10% mica
100% mica
Particle shape effects

$$V_S = \alpha \left( \frac{\sigma_x' + \sigma_y'}{2 \text{kPa}} \right)^{\beta}$$

$$\beta = 0.36 - \frac{\alpha}{700}$$

**Platy particles**

- $\frac{(R+S)}{2} < 0.55$
- $\frac{(R+S)}{2} > 0.55$

**Roundness + Sphericity**

- $\sigma_{x} + \sigma_{y}$
- $\gamma$
Drying – Unsaturated media

Shear wave velocity [m/s]

Degree of saturation $S$

glass beads + kaolinite
Stress change and cementation

Load (no unloading)

Unload and reload
## Strain regimes

<table>
<thead>
<tr>
<th></th>
<th>Small Strain</th>
<th>Large Strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deformation</td>
<td>at contacts</td>
<td>fabric changes</td>
</tr>
<tr>
<td>Stiffness</td>
<td>maximum</td>
<td>decreases</td>
</tr>
<tr>
<td>Losses</td>
<td>very low</td>
<td>large - frictional</td>
</tr>
<tr>
<td>Volume Change</td>
<td>minimal</td>
<td>potentially large</td>
</tr>
<tr>
<td>Diagenetic effects</td>
<td>potentially high</td>
<td>small in drained shear</td>
</tr>
<tr>
<td>Fabric</td>
<td>constant</td>
<td>changes towards CS</td>
</tr>
</tbody>
</table>
Threshold strains - Ranges

Threshold Strain [ ]

10^{-1} 10^{-2} 10^{-3} 10^{-4} 10^{-5} 10^{-6} 10^{-7} 10^{-8} 10^{-9}

Particle Size [m]

\[ \gamma_{dt} = 1.2 \frac{\xi}{d} \]

\[ \delta = 10^{-10} \text{ m} \quad \delta = 10^{-8} \text{ m} \]

\[ \gamma_{dt} = 1.3 \left( \frac{\sigma'}{G_g} \right)^{2/3} \]
Stress-dependent strength and $\varepsilon_{vol}$

Chain buckling: coordination $\uparrow$

Rotational frustration: coordination $\downarrow$

![Diagram of stress-dependent properties]

Graph showing deviatoric stress vs. vertical strain:
- Deviatoric Stress [kPa]
- Vertical strain [%]

Key:
- Red dashed line: Chain buckling
- Green dashed line: Rotational frustration

Legend:
- Red circles indicate stress concentrations.
Effect of roundness on $\Phi_{cv}$

$$\phi_{cv} = 42 - 17 \cdot R$$
## Evolution of internal micro-scale – 3D

<table>
<thead>
<tr>
<th>Isotropic confine.</th>
<th>At peak dev. load</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>AC (b=0)</td>
<td>AE (b=1)</td>
</tr>
<tr>
<td>Contact normals</td>
<td><img src="image" alt="Contact normals AC" /></td>
<td><img src="image" alt="Contact normals AE" /></td>
</tr>
<tr>
<td>$\mathbf{N}(\theta)$</td>
<td><img src="image" alt="N(\theta) AC" /></td>
<td><img src="image" alt="N(\theta) AE" /></td>
</tr>
<tr>
<td>$\mathbf{T}(\theta)$</td>
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<td><img src="image" alt="T(\theta) AE" /></td>
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</table>

(magnified x5)

Chantawarangul, 1993
Constant angle of repose?

\[ \phi_{\text{int}} = 1.5 \phi_{\text{ext}} \]

Narsilio, Dodds, Fugle, Trott, Kim, Yun
Anisotropy in $\phi$ - Clays

$\phi_E = 1.5 \phi_C$

$\phi_E = \phi_C$

Mayne & Holtz 1985
Undrained strength anisotropy

*Controlled by the generation of pore pressure*
- *chain buckling and skeletal stiffness*
- *spatial variability of e*
Undrained strength anisotropy

Controlled by the generation of pore pressure
- chain buckling and skeletal stiffness
- spatial variability of $e$
- threshold strain

Mayne and Holtz 1985

Clays

Mayne and Holtz 1985
Undrained strength anisotropy

Controlled by the generation of pore pressure
- chain buckling and skeletal stiffness
- spatial variability of e
- threshold strain
- fabric anisotropy

Fraser River Sand

AC: \( b=0 \ \alpha=0 \)

SS: \( b>0 \ \alpha>0 \)

AE: \( b=1 \ \alpha=90 \)

Vaid and Sivathayalan 1996
# Scales

<table>
<thead>
<tr>
<th>temporal</th>
<th>spatial</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{t_{\text{load}}}{L^2/c_v}$</td>
<td>draining or undrained</td>
</tr>
<tr>
<td>$\frac{f}{f_{\text{res}}}$</td>
<td>quasi-static or dynamic</td>
</tr>
<tr>
<td>$\frac{L/ki}{L^2/c_v}$</td>
<td>seepage or consolidation</td>
</tr>
</tbody>
</table>

![Graph](image1.png)

![Graph](image2.png)
soil

- fluid
- particles
  - mineral
    - surface charge
    - double layers
    - capillarity
  - size
    - forces
    - specific surface
    - relative size
  - shape
    - sphericity
    - roundness
    - smoothness
- microorganisms
  - fine-grained soil fabric
  - coarse-grained soil packing
  - platy particles

- conduction
- diffusion
- energy coupling
- stiffness
- threshold strain
- strength
  - (scales)
- rheology
- diagenesis
<table>
<thead>
<tr>
<th>Sub-class</th>
<th>Gravel:</th>
<th>Sand:</th>
<th>Mineral, $S_s$</th>
<th>Pore fluid</th>
</tr>
</thead>
<tbody>
<tr>
<td>COARSE</td>
<td>&gt; 50% retained sieve #4</td>
<td>&gt; 12% fines</td>
<td>LL&lt;50</td>
<td>CH</td>
</tr>
<tr>
<td>&gt; 50% retained sieve #200</td>
<td>&gt; 5% fines</td>
<td>LL&gt;50</td>
<td>OL or ML</td>
<td>OH or MH</td>
</tr>
<tr>
<td>FINE</td>
<td>LL&lt;50</td>
<td>&lt; 5% fines</td>
<td>CL-ML</td>
<td>CL</td>
</tr>
<tr>
<td>&lt; 50% retained sieve #200</td>
<td>LL&gt;50</td>
<td>&gt; 12% fines</td>
<td>CL</td>
<td>OH</td>
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Legend:
- **relative size**
  - GW
  - GP
  - GM
  - GC
  - SW
  - SP
  - SM
  - SC
- **filled porosity**
  - ML
  - CL
  - OL
  - MH
  - CH
  - OH

- **50-100 μm**

- **mineral, $S_s$**

- **pore fluid**

- **plot:**
  - Relative size:
    - COARSE: GW, GP
    - FINE: SW, SP, SM, SC
  - Filled porosity:
    - GW: GM, GC
    - SW: SM, SC
  - Pore fluid:
    - GW: OL, MH
    - SW: ML, CL
    - SP: CL, ML
    - SM: CH, OH
    - SC: CH, OH
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<td></td>
</tr>
<tr>
<td>LL&lt;50</td>
<td>LL&gt;50</td>
<td></td>
<td></td>
</tr>
<tr>
<td>60</td>
<td>50</td>
<td>40</td>
<td>30</td>
</tr>
<tr>
<td>pore fluid (explicitly)</td>
<td>degree of saturation</td>
<td>specific surface - coarse</td>
<td>extent of diagenesis</td>
</tr>
</tbody>
</table>

- **Gravel:**
  - > 50% retained sieve #200
  - LL<50

- **Sand:**
  - < 50% retained sieve #4
  - LL>50

- **Specific Surface:**
  - coarse< 5% fines\
  - fines (explicitly)

- **Liquid Limit:**
  - LL<50
  - LL>50

- **Plasticity Index:**
  - 60
  - 50
  - 40
  - 30
  - 20
  - 10
  - 0

- **C_{u>4}, 1 \leq C_{c} \leq 3**
- **C_{u>6}, 1 \leq C_{c} \leq 3**

- **Degree of Saturation:**
  - GW
  - GP
  - GM
  - GC
  - SW
  - SP
  - SM
  - SC

- **Pore Fluid:**
  - GW
  - GP

- **Gravels:**
  - GW
  - GP

- **Fines:**
  - GM
  - GC

- **Sand:**
  - GW
  - GP

- **Anisotropy:**
  - ML
  - CL
  - OL
  - MH
  - CH
  - OH

- **Diagenesis:**
  - ML
  - CL
  - OL
  - MH
  - CH
  - OH

- **Bio-activity and Bio-viability:**
  - ML
  - CL
  - OL
  - MH
  - CH
  - OH