

Innovations, Challenges, and Future Opportunities

**J. Carlos Santamarina
Georgia Institute of Technology**

electromagnetic waves

Maxwell's Equations

Gauss' Law of Electricity

$$\int_{\text{surf}} \epsilon \mathbf{E} \cdot d\mathbf{s} = \int_{\text{vol}} \rho_v^{<\text{free}>} dv$$

$$\nabla \cdot \mathbf{E} = \frac{1}{\epsilon} \rho_v^{<\text{free}>}$$

Gauss' Law of Magnetism

$$\int_{\text{surf}} \mathbf{H} \cdot d\mathbf{s} = 0$$

$$\nabla \cdot \mathbf{H} = 0$$

Faraday's Law of Induction

$$\int_{\text{loop}} \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt} \int_{\text{surf}} \mu \mathbf{H} \cdot d\mathbf{s}$$

$$\nabla \times \mathbf{E} = -\mu \frac{d\mathbf{H}}{dt}$$

Ampere-Maxwell's Law

$$\int_{\text{loop}} \mathbf{H} \cdot d\mathbf{l} = \int_{\text{surf}} \mathbf{J} \cdot d\mathbf{s} + \frac{d}{dt} \int_{\text{surf}} \epsilon \mathbf{E} \cdot d\mathbf{s}$$

$$\nabla \times \mathbf{H} = \sigma \mathbf{E} + \epsilon \frac{d\mathbf{E}}{dt}$$

Wave Equation

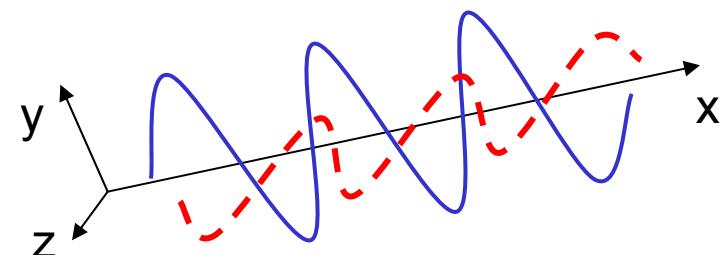
$$\nabla^2 \mathbf{E} = \mu^* \sigma \frac{\partial \mathbf{E}}{\partial t} + \mu^* \epsilon^* \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

Solution: $E_y = E_o e^{-\alpha x} e^{j(\omega t - \kappa x)} = E_o e^{j(\omega t - \gamma^* x)}$

Then

$$\gamma^* = \alpha + j\kappa = \sqrt{j\omega \sigma \mu^* - \omega^2 \epsilon^* \mu^*}$$

Faraday: $H_z = -j \frac{\gamma^*}{\mu \omega} E_y$



Phase Velocity

$$V_{ph} = \frac{\omega}{\text{Im}(\gamma^*)} = \frac{\omega}{\text{Im}\left(\sqrt{j\omega\sigma\mu^* - \omega^2\epsilon^*\mu^*}\right)}$$

non-ferromagnetic / dielectric

$$\mu^* = \mu_0 \quad \epsilon^* = \epsilon' \quad \sigma = 0$$

$$V_{ph} = \frac{c_0}{\sqrt{\epsilon'/\epsilon_0}}$$

Attenuation

$$\alpha = \text{Re}(\gamma^*) = \text{Re}\left(\sqrt{j\omega\sigma\mu^* - \omega^2\epsilon^*\mu^*}\right)$$

non-ferromagnetic

$$\mu^* = \mu_0 \quad \epsilon^* = \epsilon' + j\epsilon'' \quad \sigma$$

$$\alpha = \frac{\omega \sqrt{\epsilon'/\epsilon_0}}{c_0} \sqrt{\frac{1}{2} \left(\sqrt{1 + \tan^2 \delta} - 1 \right)}$$

For 1D propagation

Skin depth $S_d = \frac{1}{\alpha} = \frac{1}{\text{Re}(\gamma^*)}$

Impedance $Z^* = \frac{E_y}{H_z} = j \frac{\omega}{\gamma^*} \mu^*$

Reflection and Transmission

$$R^* = \frac{1 - (z_1^* / z_2^*)}{1 + (z_1^* / z_2^*)} \quad T^* = \frac{2}{1 + (z_1^* / z_2^*)}$$

Electromagnetic Parameters

Conductivity σ

Permittivity $\epsilon^* = \epsilon' - j \epsilon''$

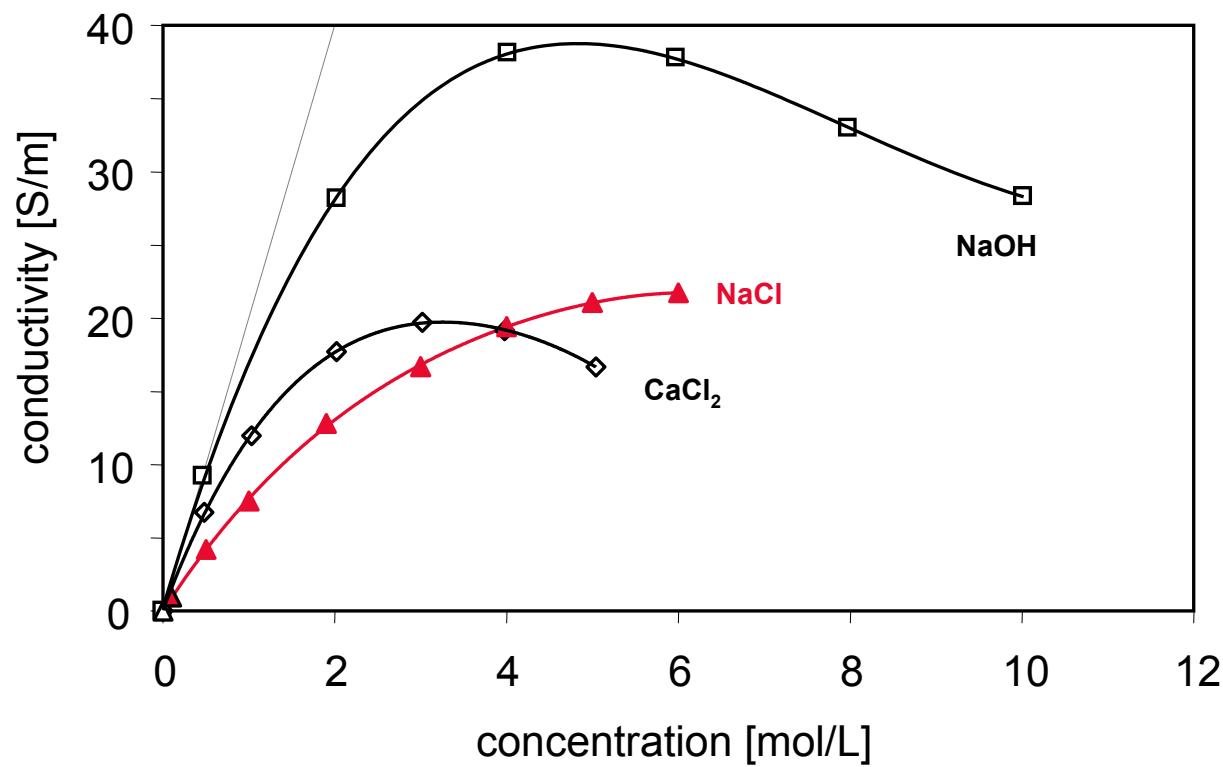
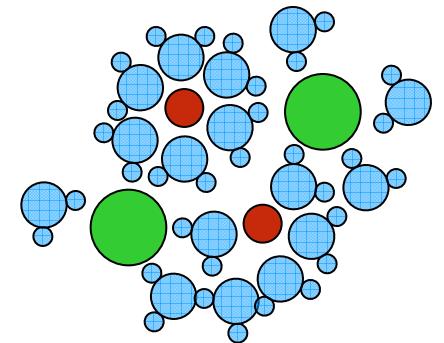
Permeability $\mu = \mu' - j \mu''$

Effective conductivity $\sigma_{\text{eff}} = \epsilon' \omega \mu_r'' + (\sigma + \epsilon'' \omega) \mu_r'$

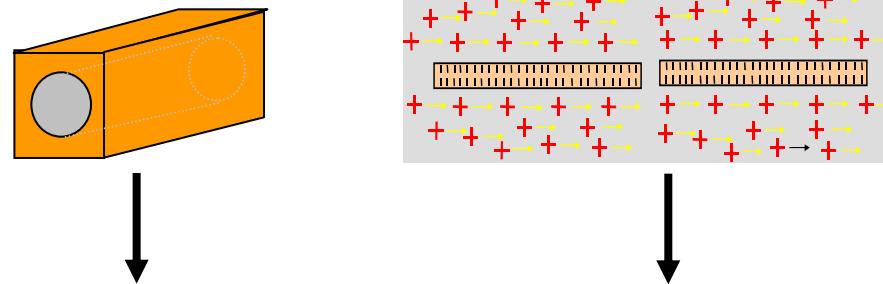
*details and references in
Santamarina, Klein and Fam
Soils and Waves – J. Wiley*

electromagnetic properties

Conductivity - Electrolytes



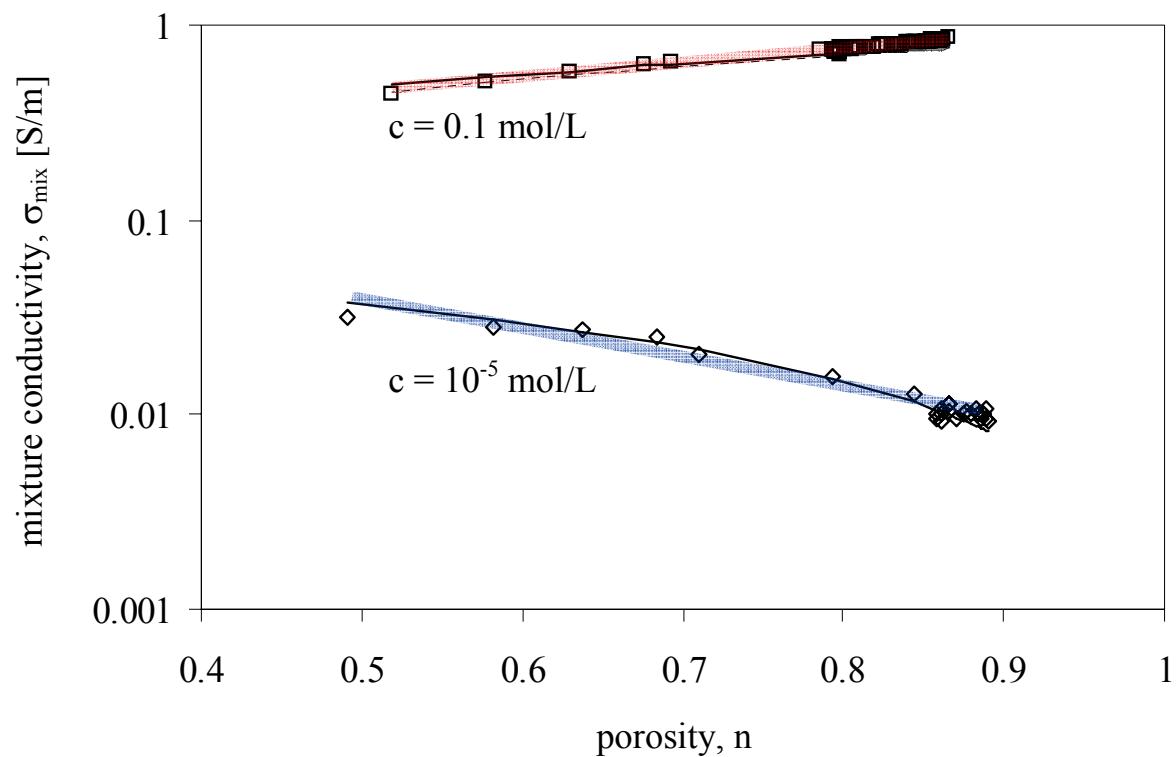
Bulk and Surface Conduction



$$\sigma_{\text{soil}} = n \sigma_{\text{el}} + (1-n) 2\rho_p \lambda_{\text{ddl}} S_a$$

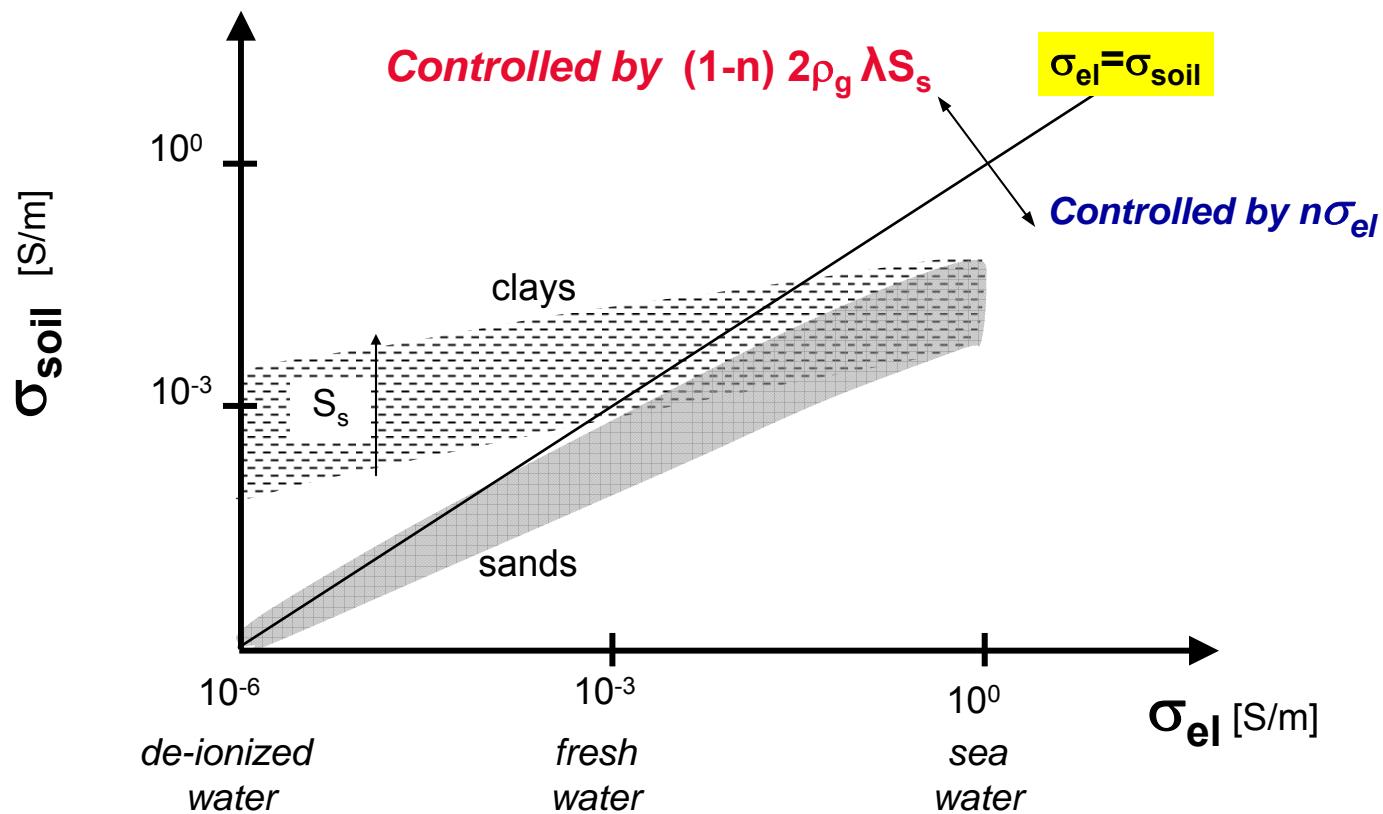
$$\sigma_{\text{soil}} = \alpha n^\beta \sigma_{\text{el}} \quad (\text{Archie})$$

Conductivity: Archie?

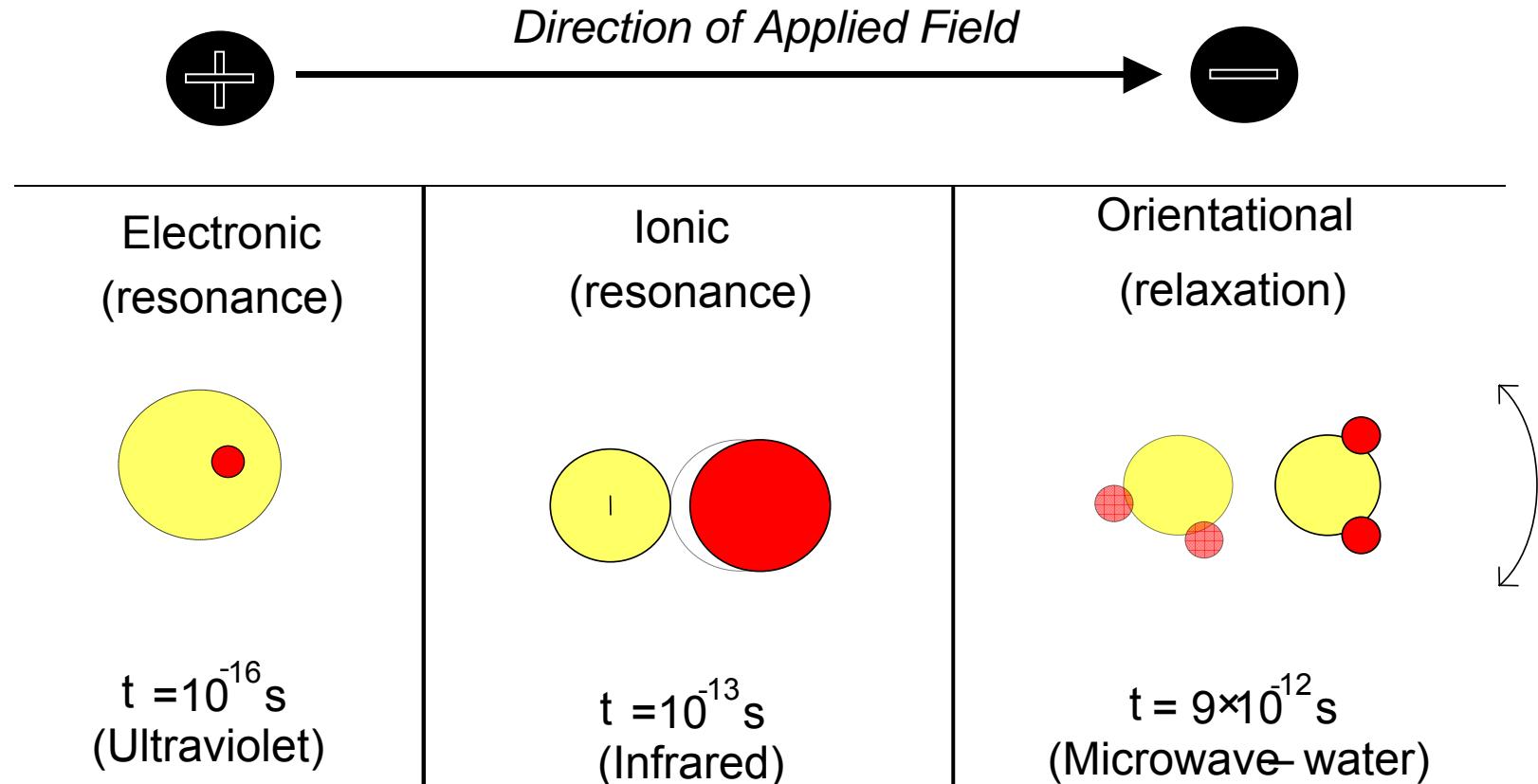


$$\sigma_{\text{soil}} = n \sigma_{\text{el}} + (1-n) 2\rho_g \lambda S_a$$

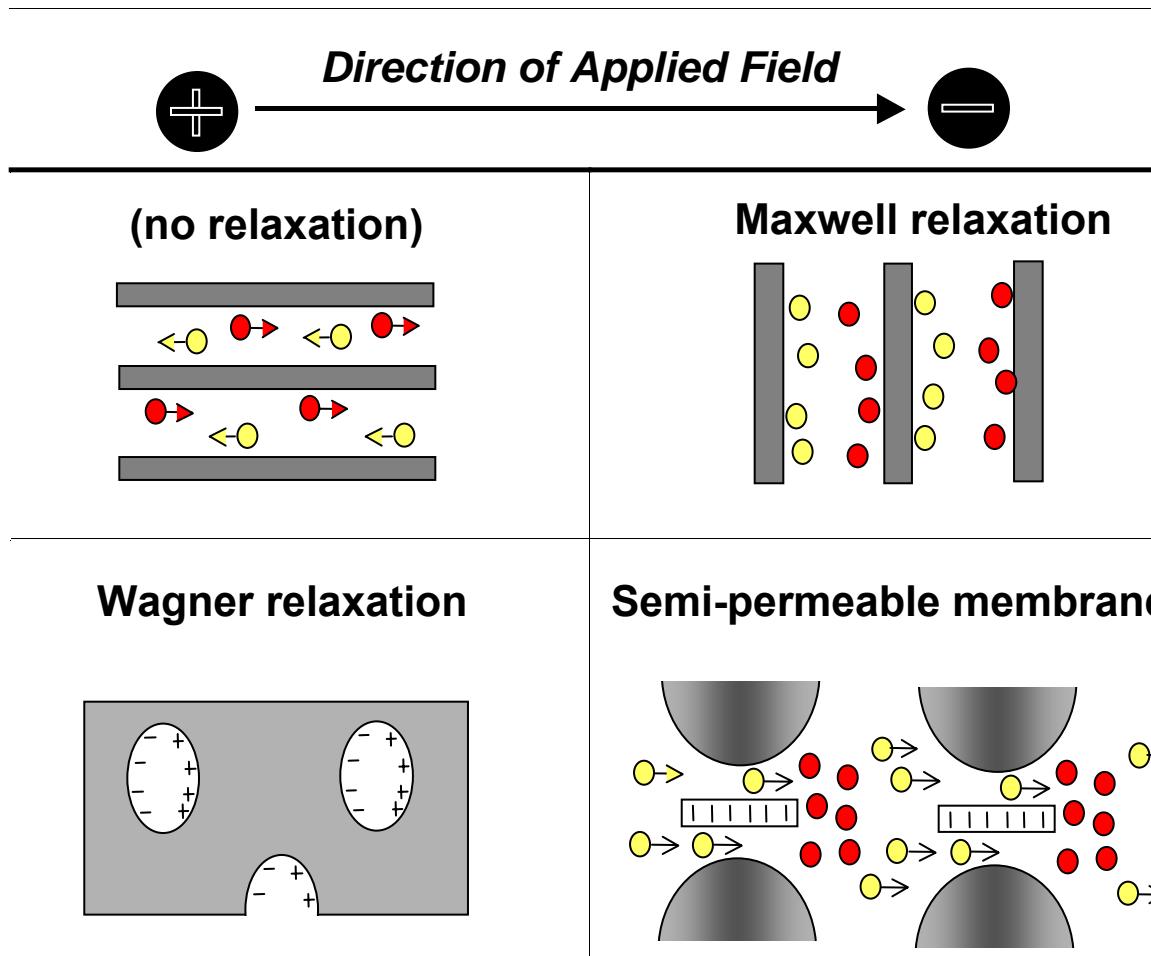
Conductivity - Summary



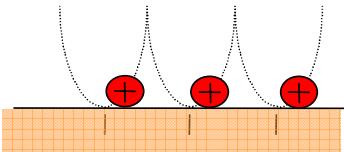
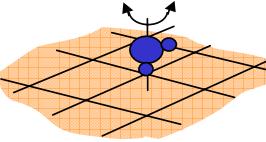
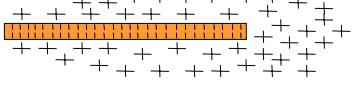
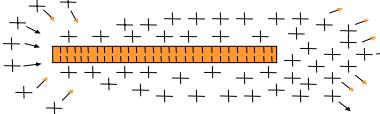
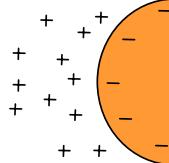
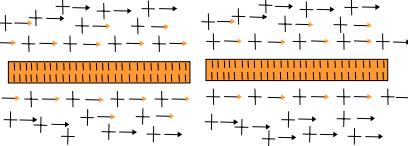
Polarization – Single phase



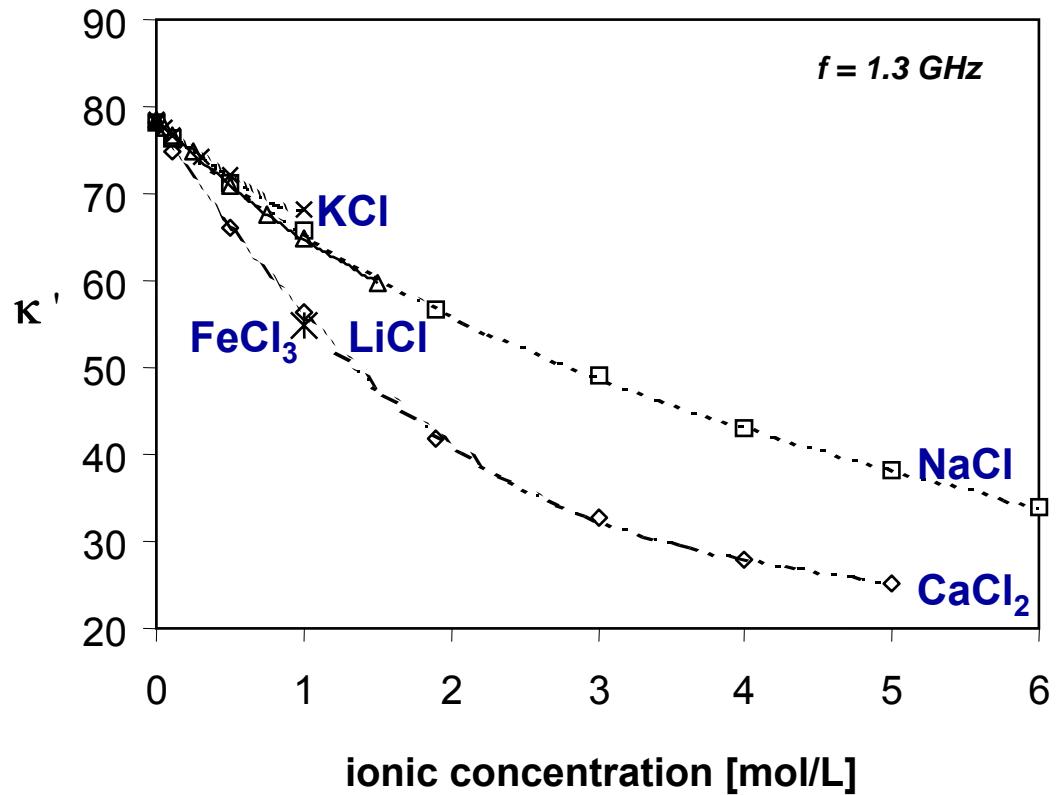
Two-phase media - Spatial polarization



Double layer effects

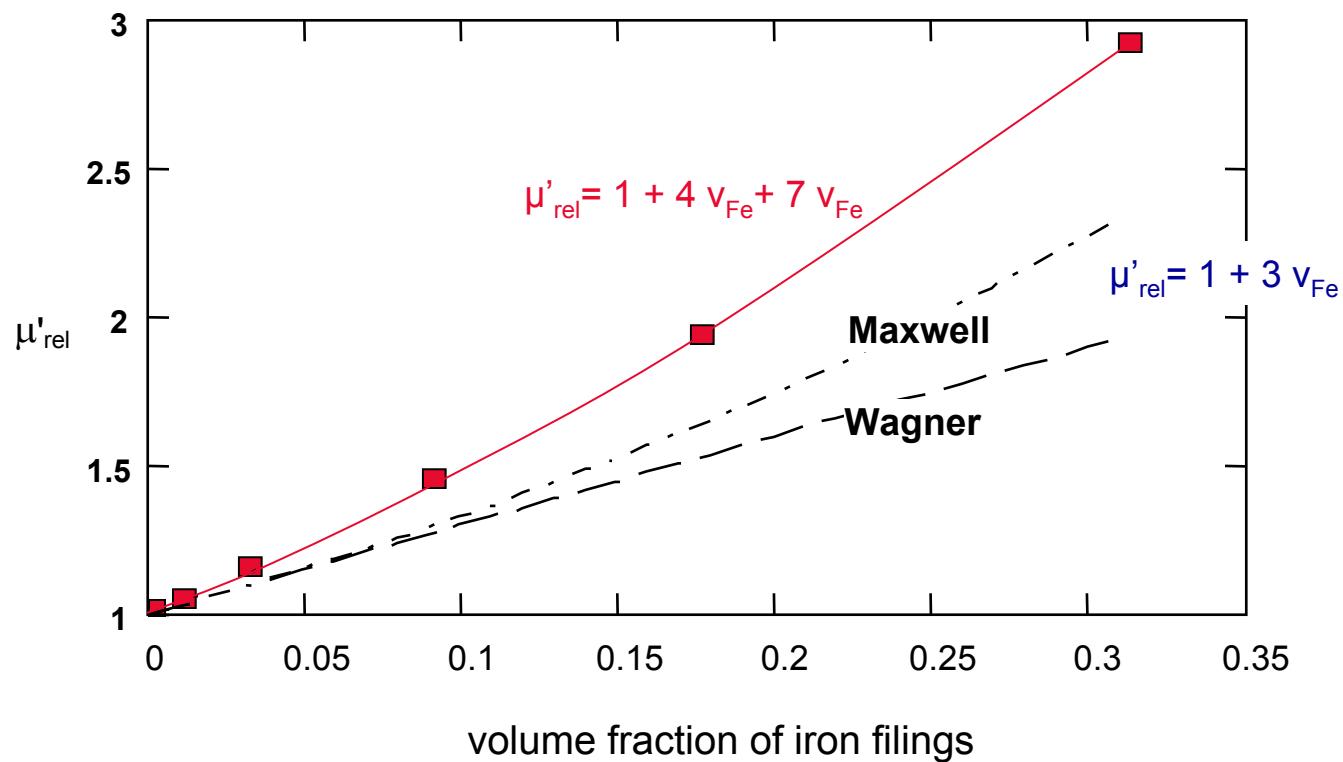
| <i>Direction of Applied Field</i> → | |
|--|--|
| Stern layer  (Infrared) | Bound water (relaxation)  (Radio frequency) |
| Double layer (deionized)  | Double layer (electrolyte)  |
| Double layer - Normal  | particle interactions (surface conduction)  |

Water-Ion Interaction



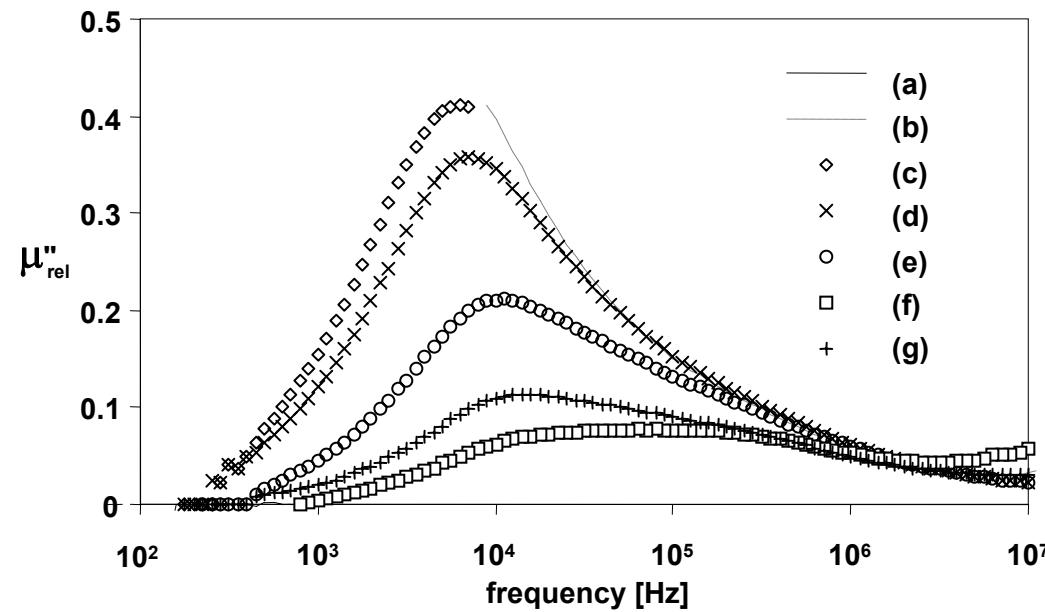
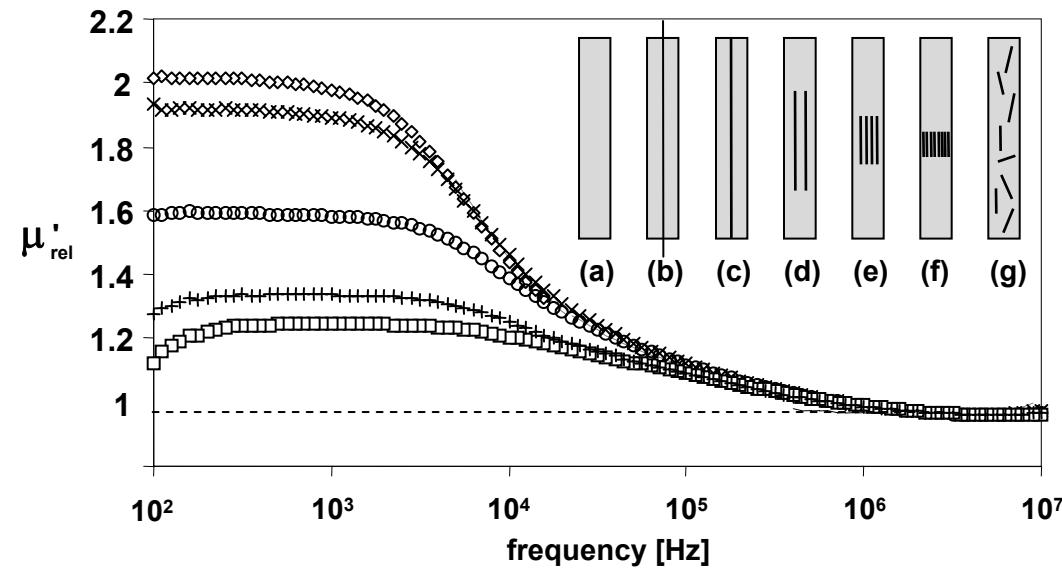
Permeability

iron fillings in kaolinite – f = 10 kHz



Permeability

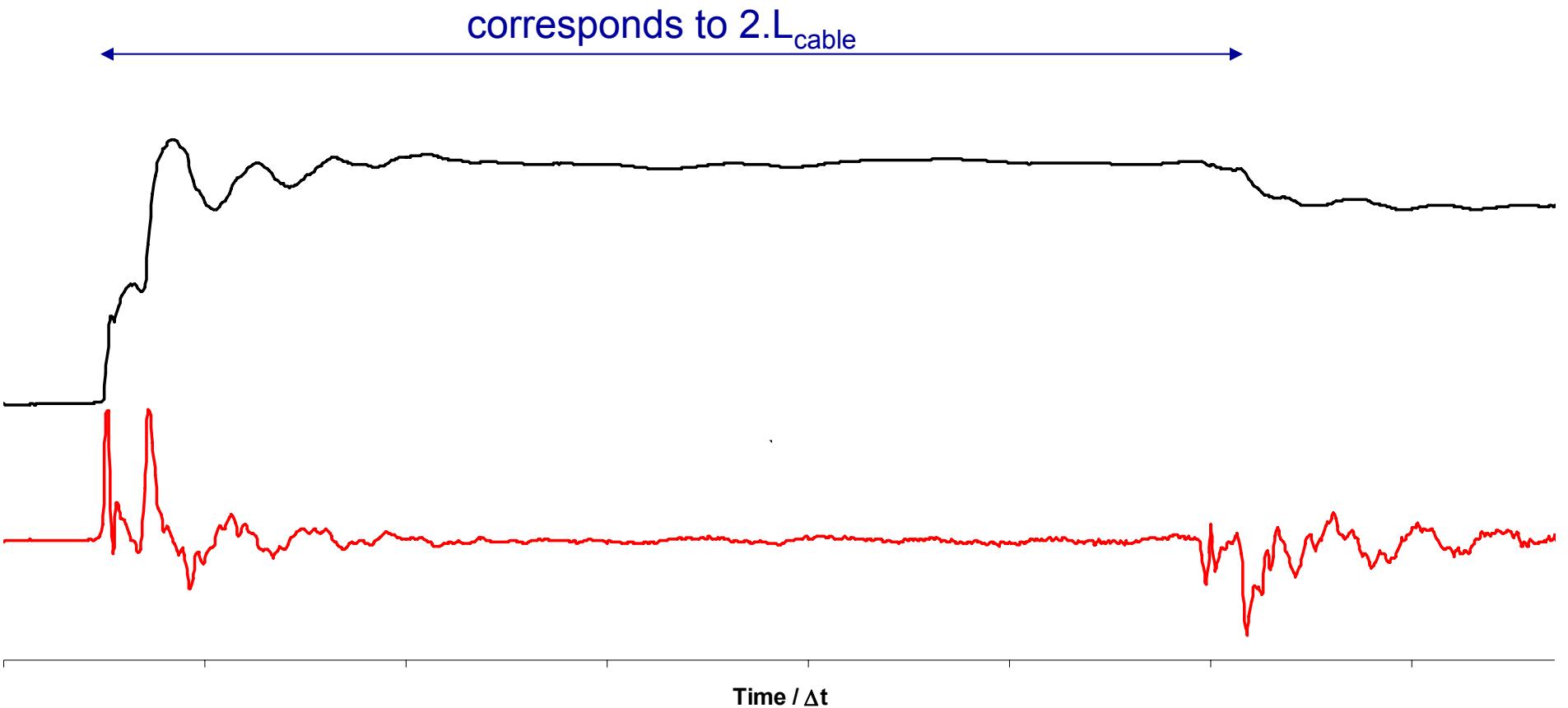
iron in kaolinite



All data by Dante Fratta (U. Wisconsin)

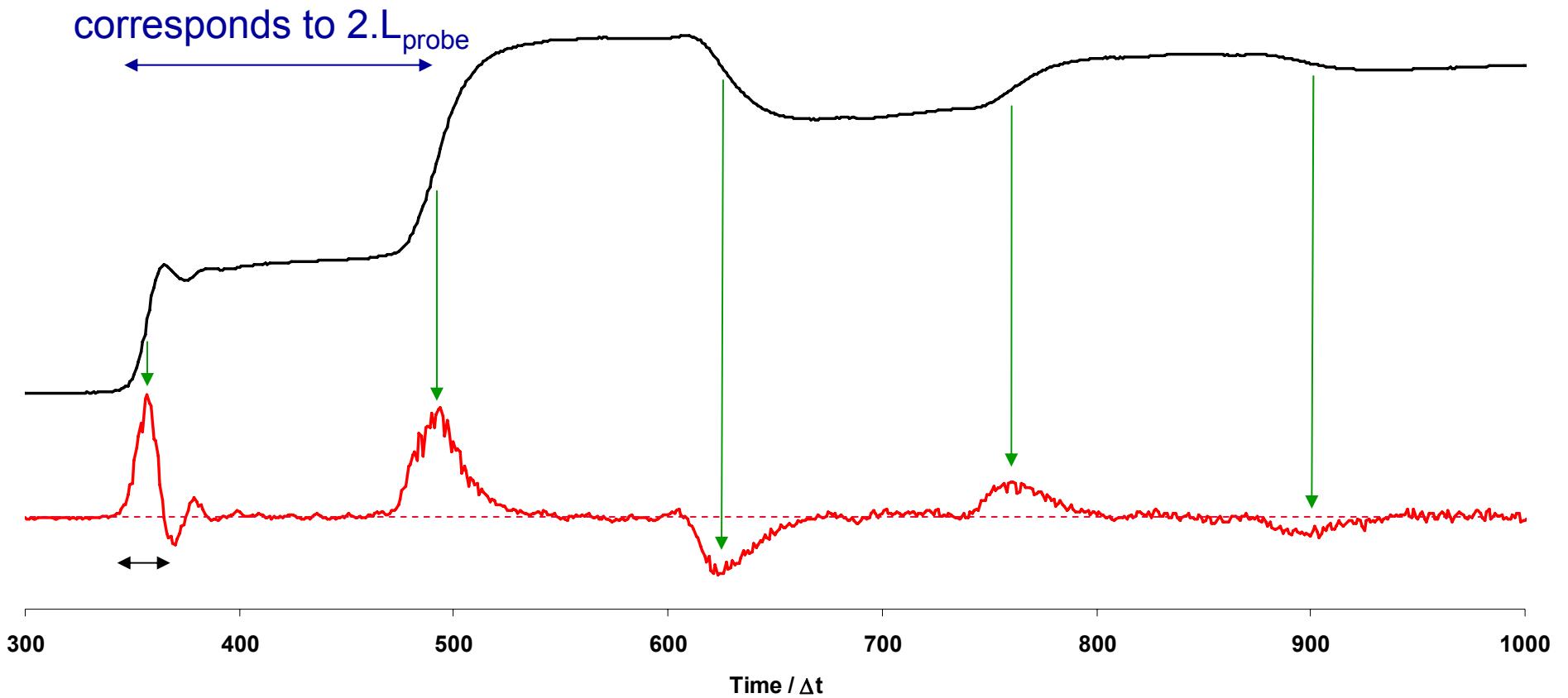
TDR measurements

The Cable



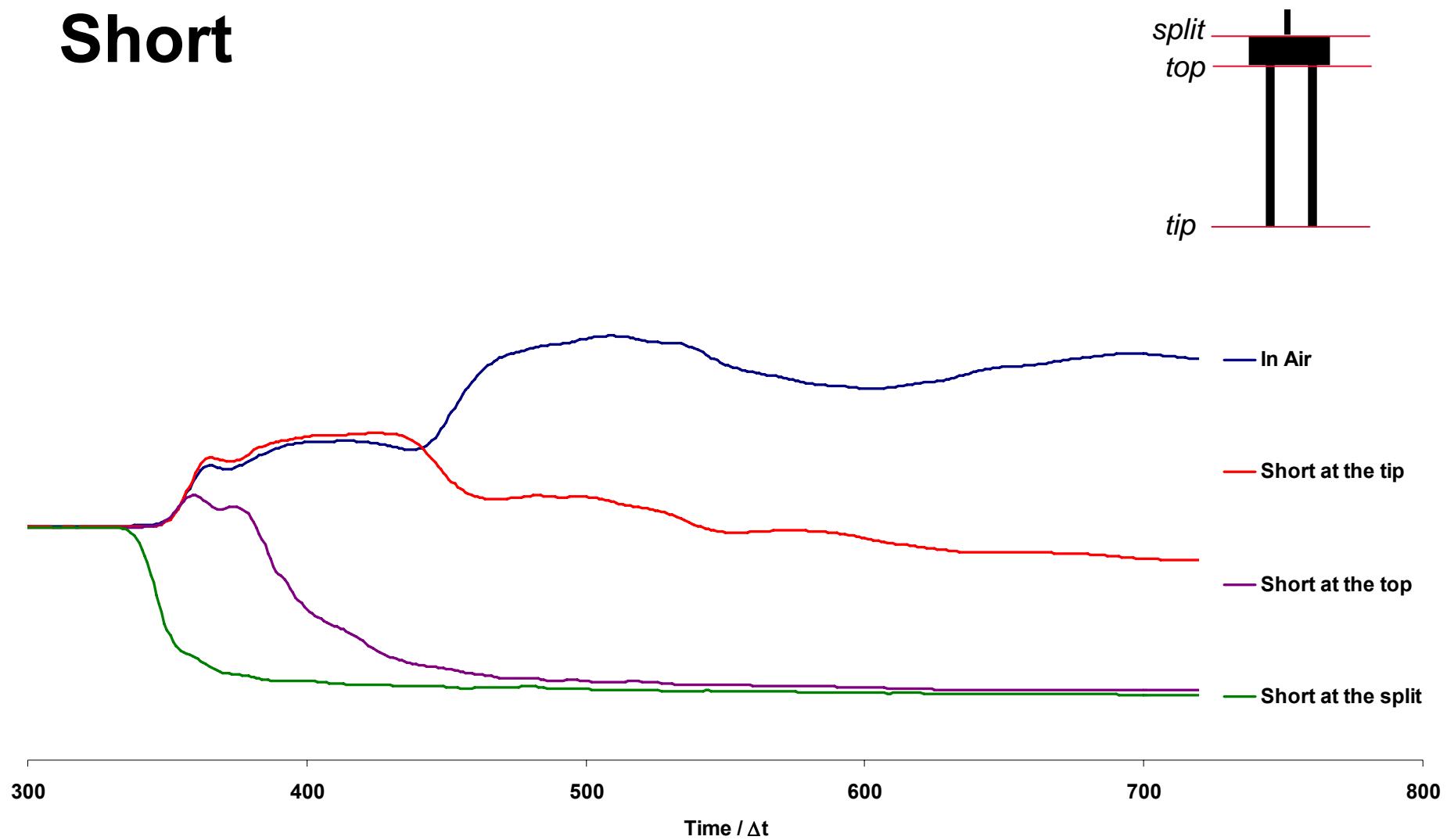
*the probe = complex end-reflector
signal changed sign at equipment*

The Probe



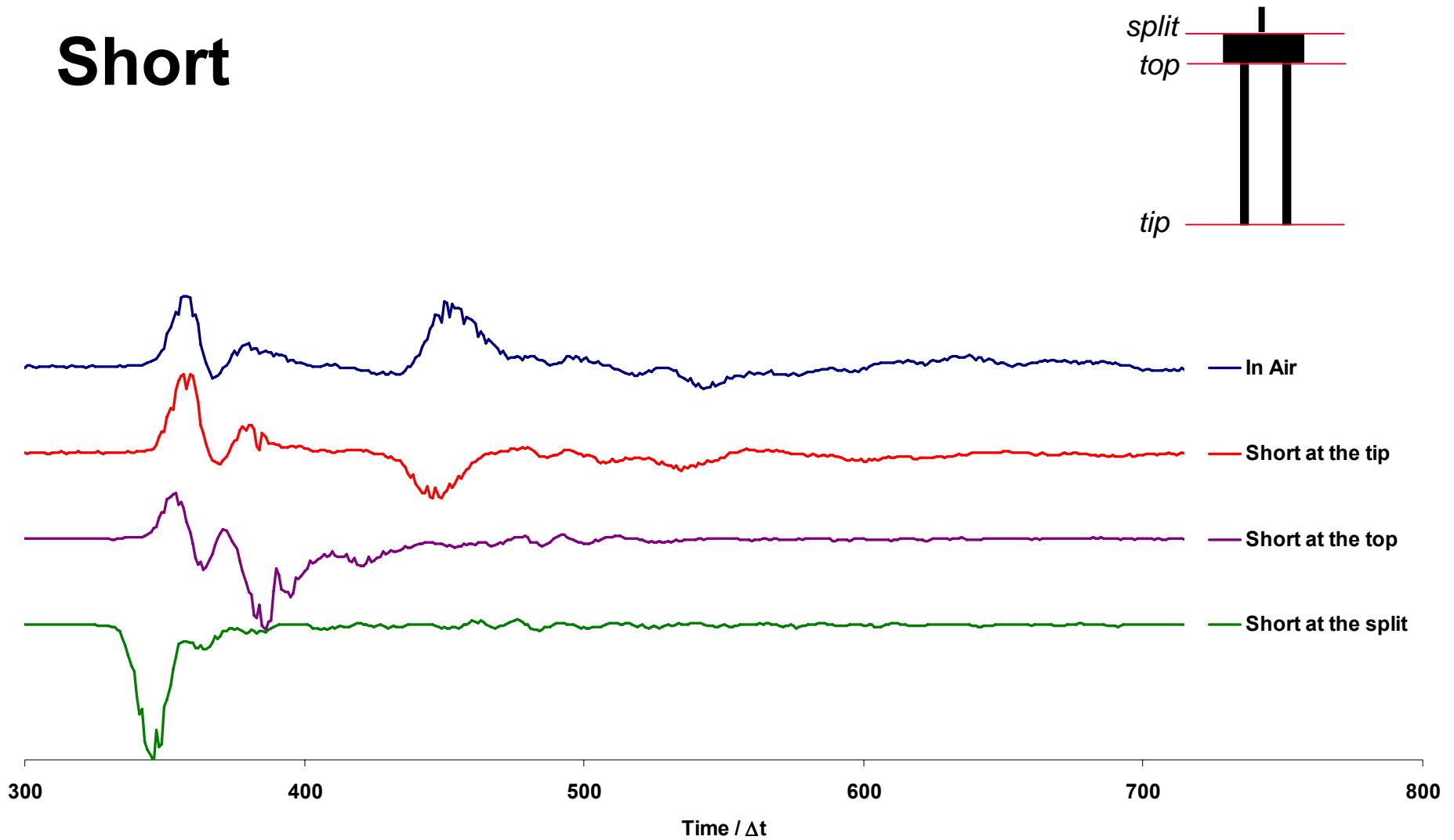
$f \sim 1$ to 3 GHz
dispersion
multiples

Short



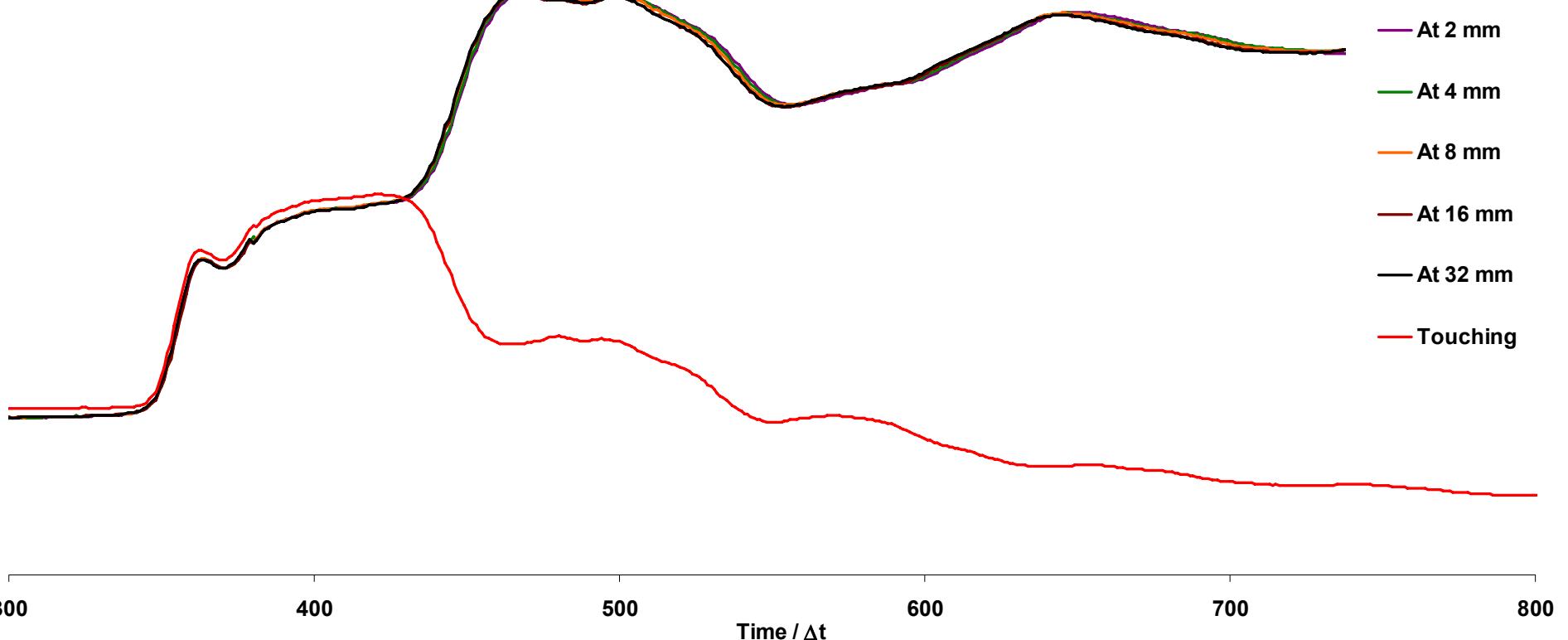
*where is zero-time?
composite reflection at top
open and short tip impedance*

Short



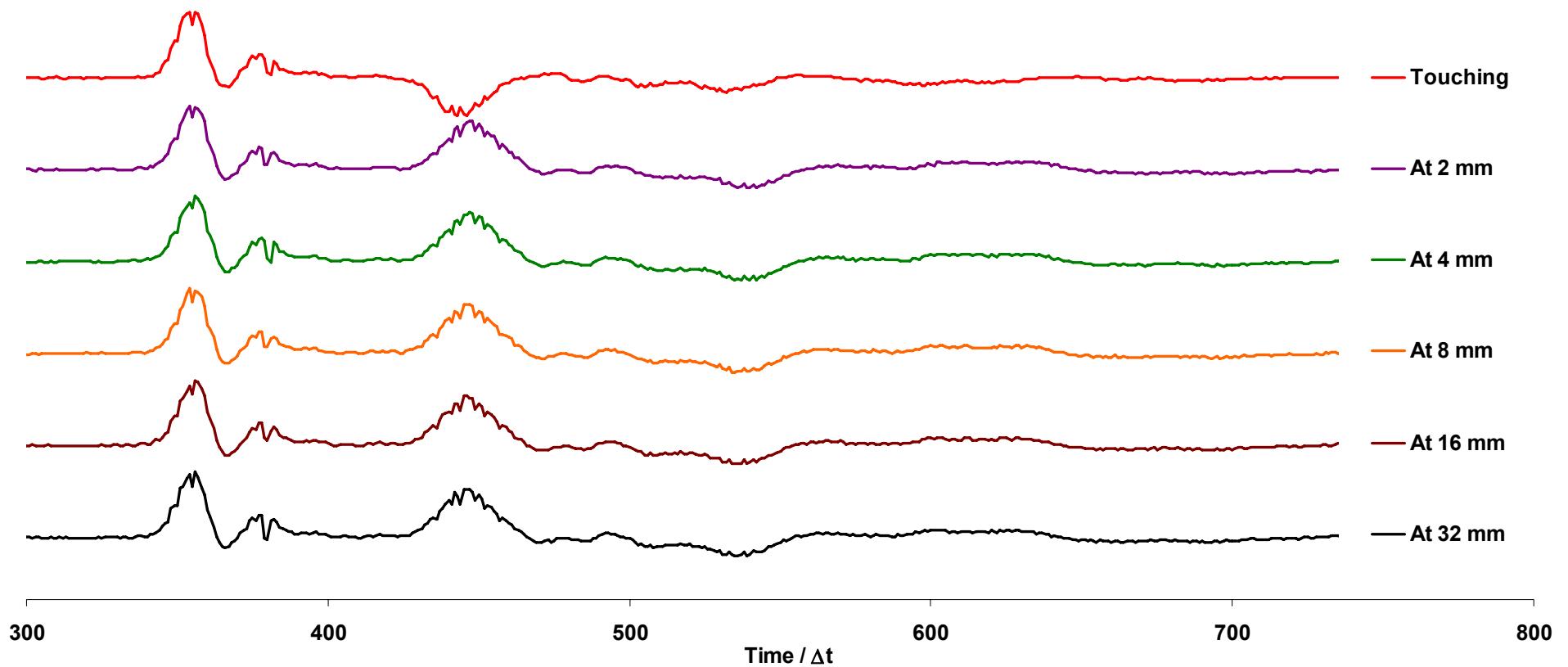
*where is zero-time?
composite reflection at top
open and short tip impedance*

Boundaries: Normal Plate



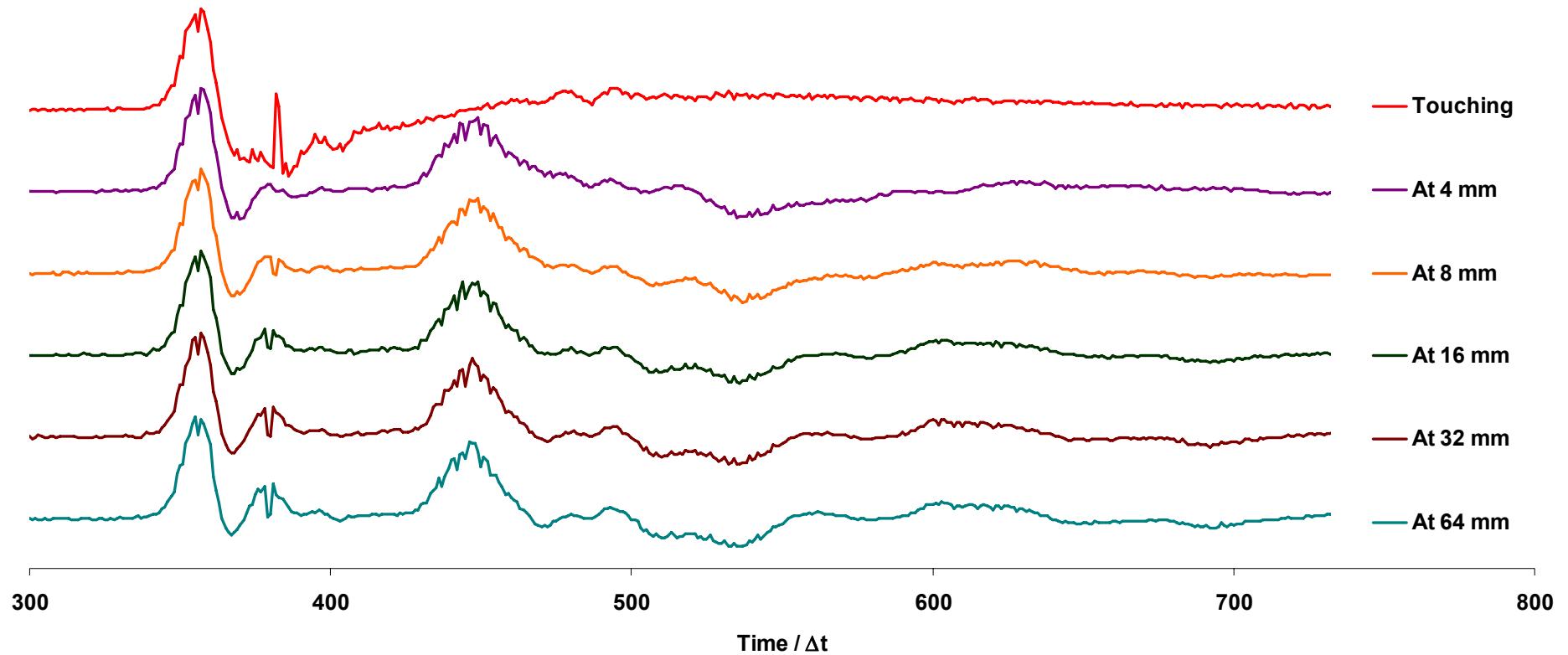
does not see ahead of tip

Boundaries: Normal Plate



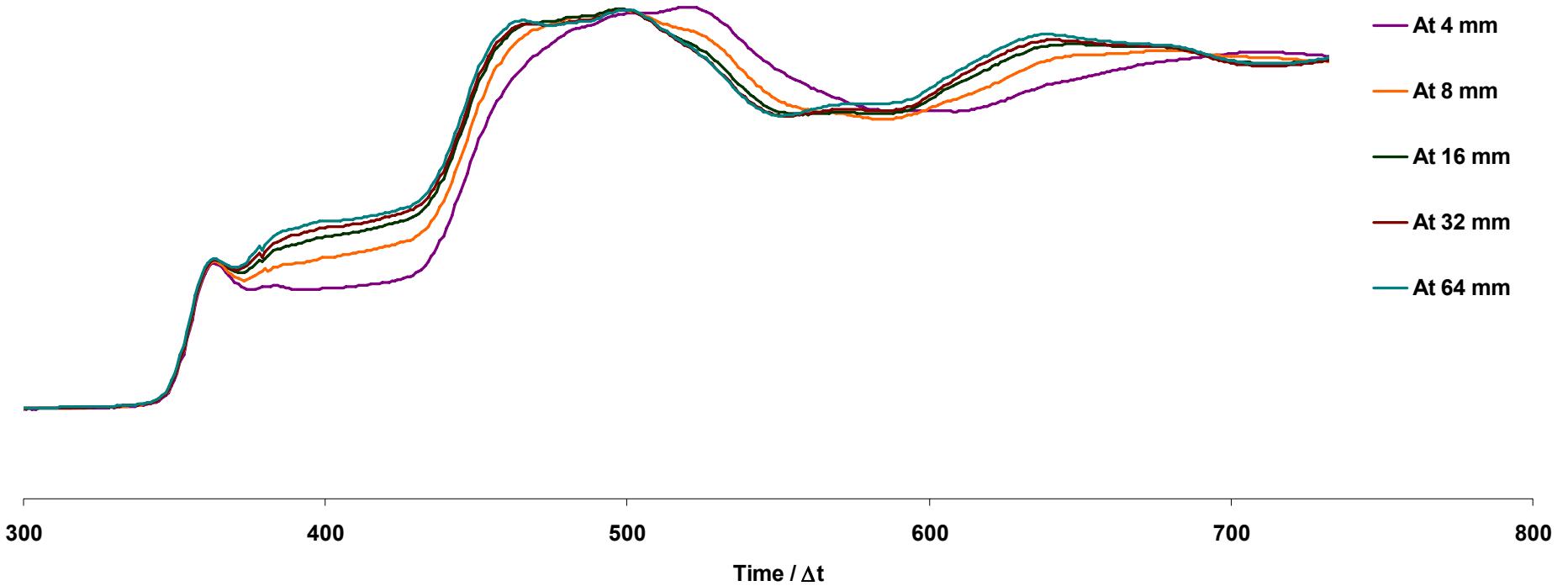
does not see ahead of tip

Boundaries: Parallel Plate



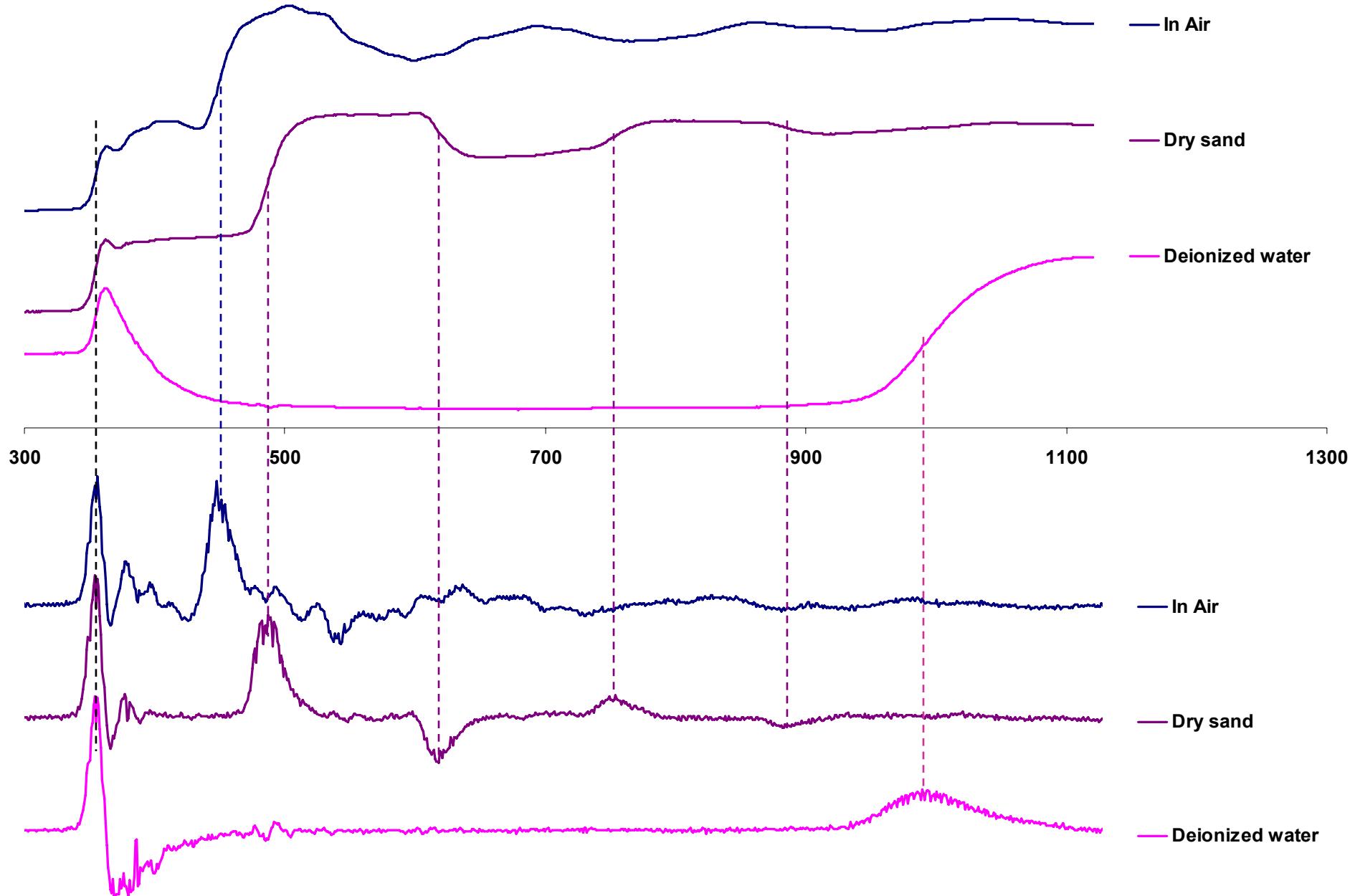
does not feel outside inter-rod ?

Boundaries: Parallel Plate



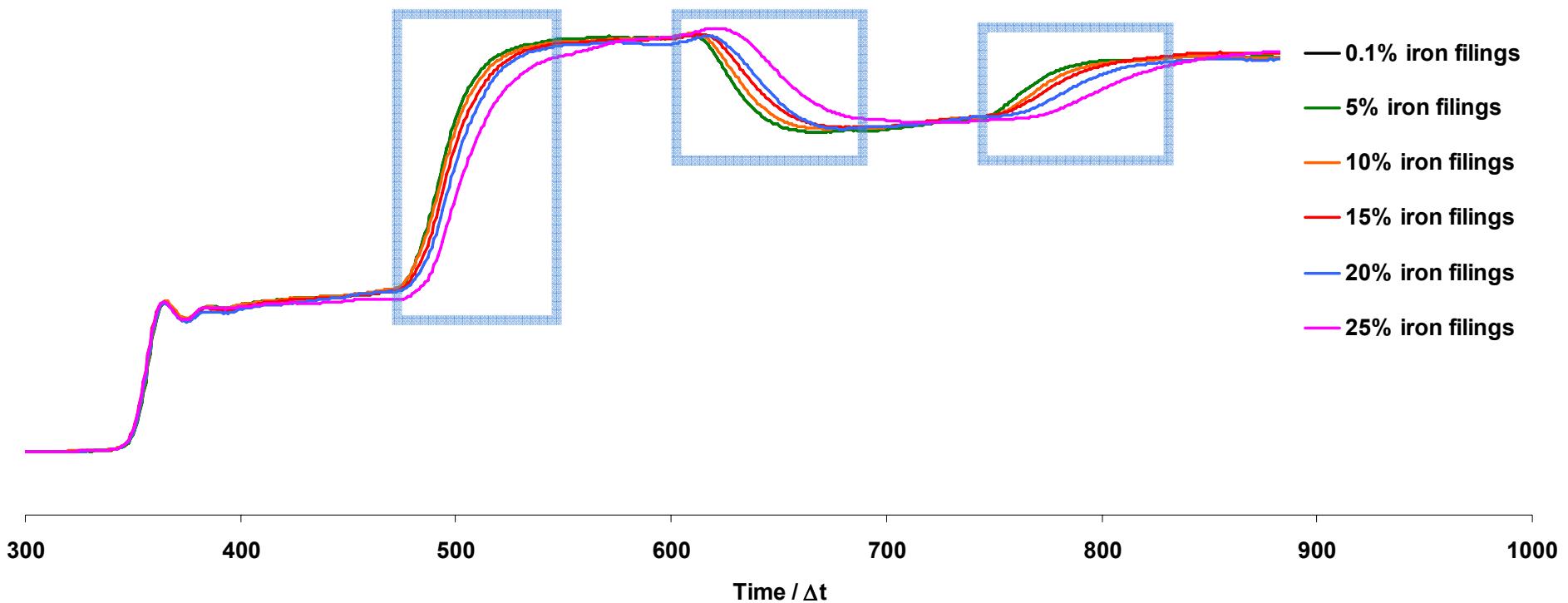
H-field effect!

Permittivity



Permeability

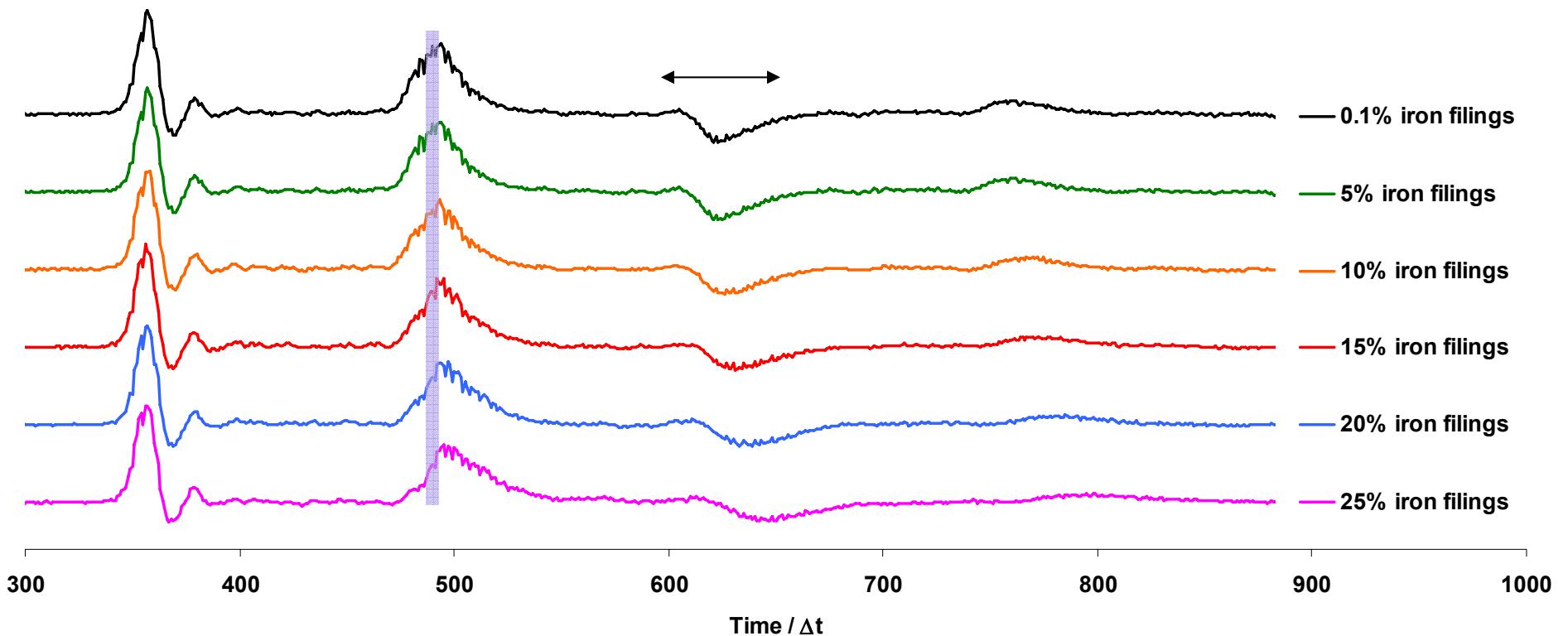
Sand +



expect minor effect

Permeability

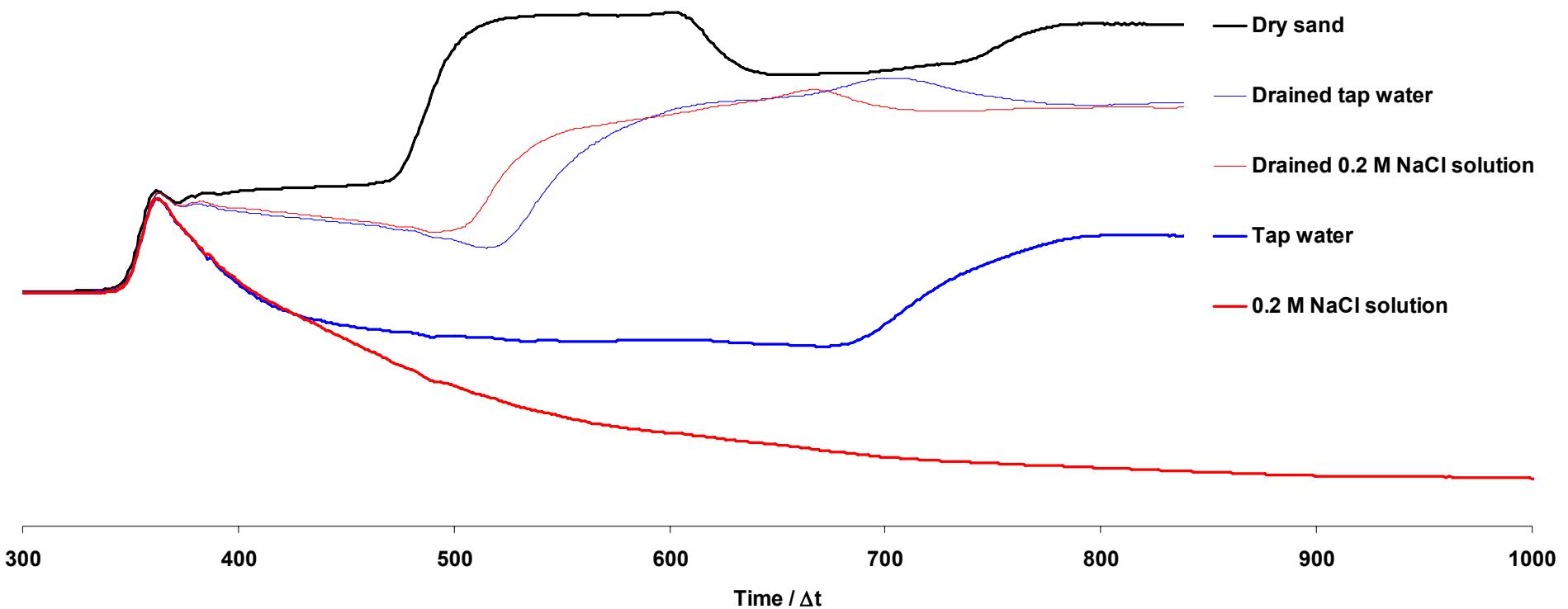
Sand +



expect minor effect

Conductivity

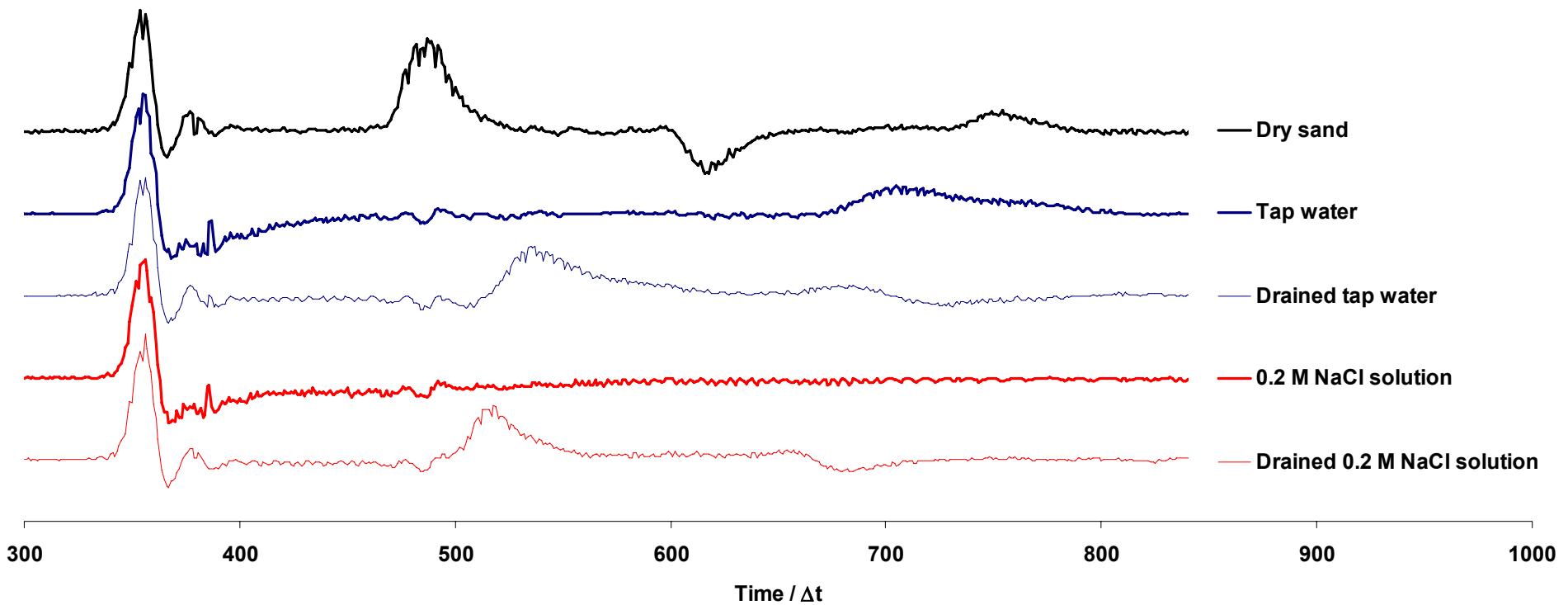
Sand



good assessment of conductivity

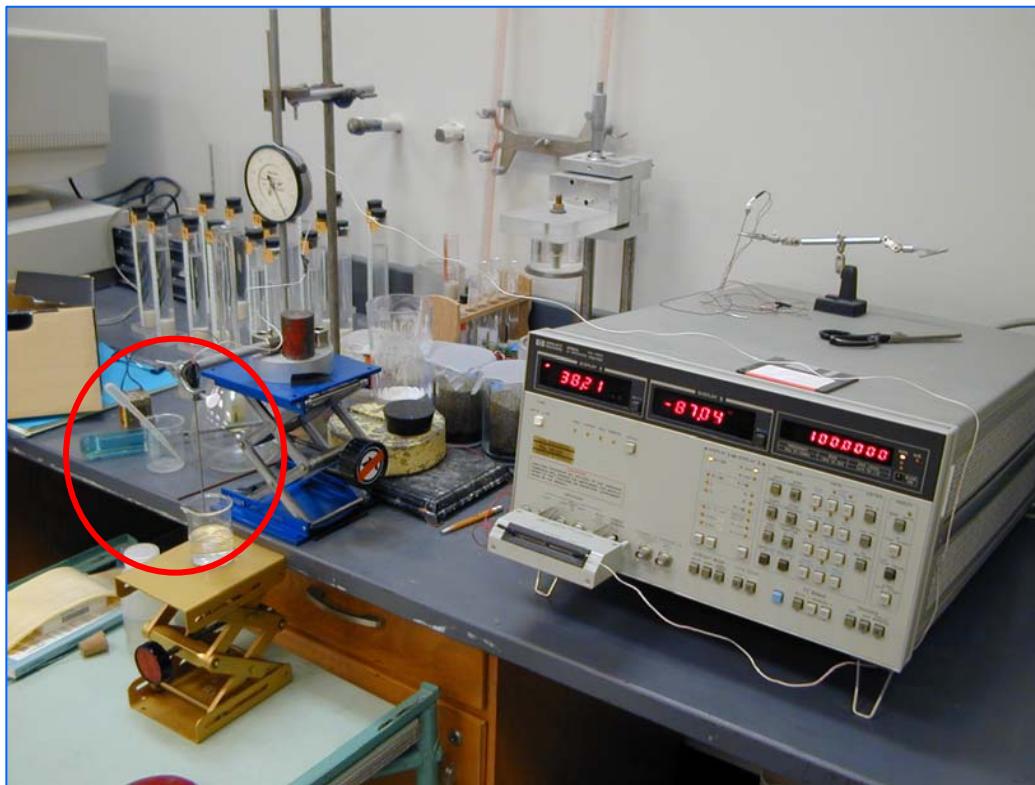
Conductivity

Sand

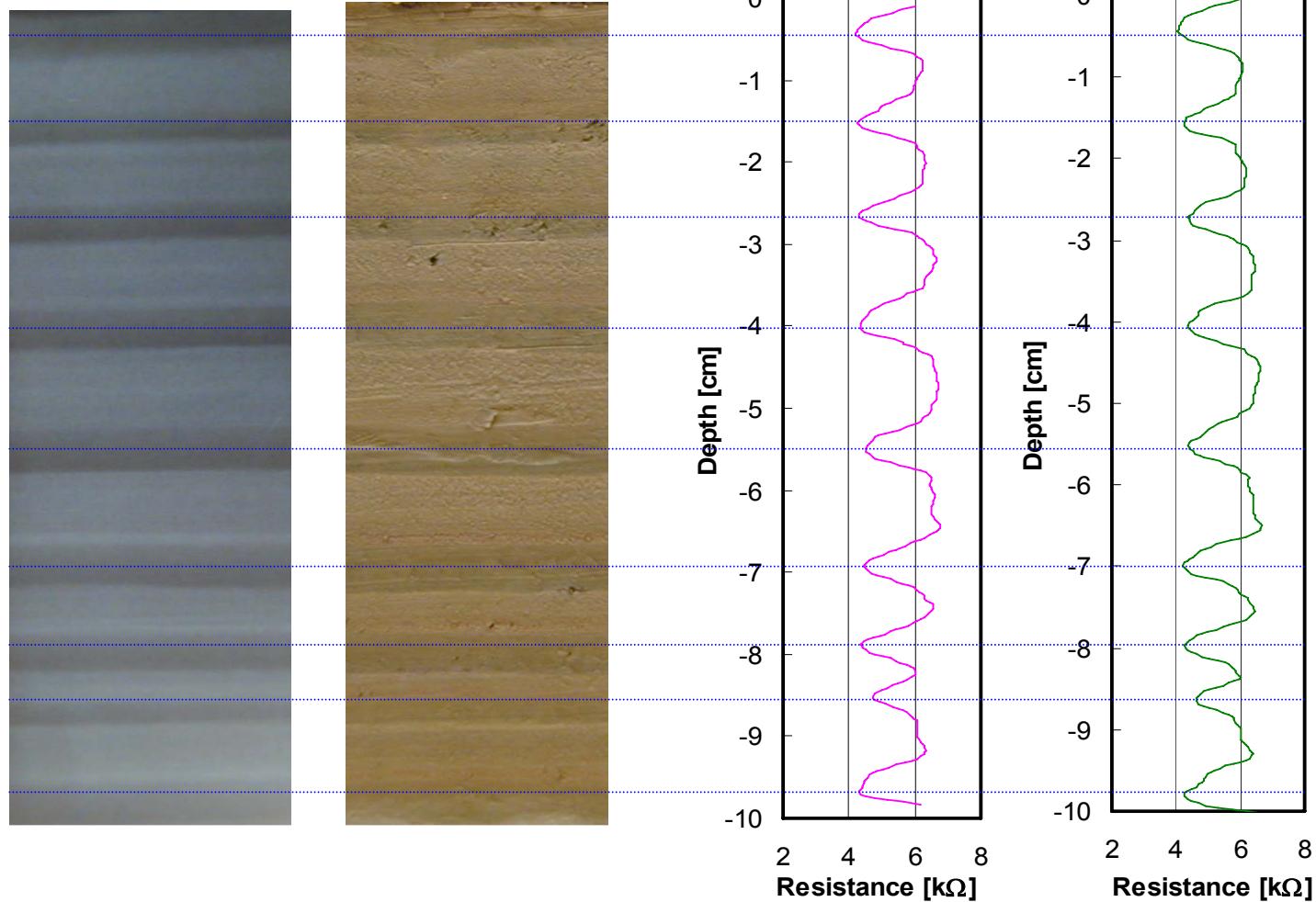


*recall skin depth
may not see tip reflection*

Heterogeneity – Layering



Varved Clay



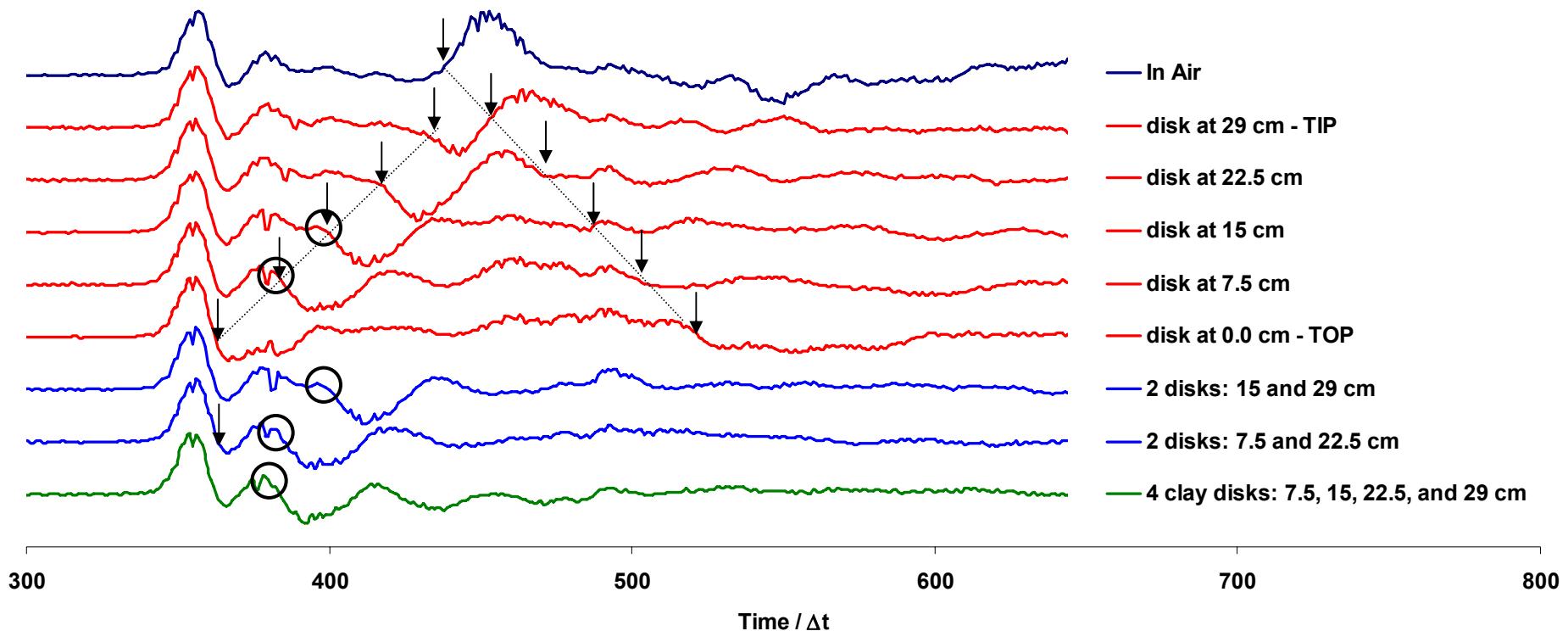
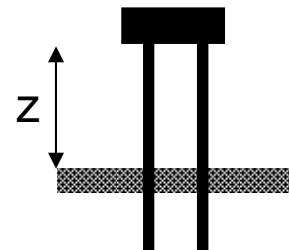
X-Ray

Photograph

Needle probe measurements

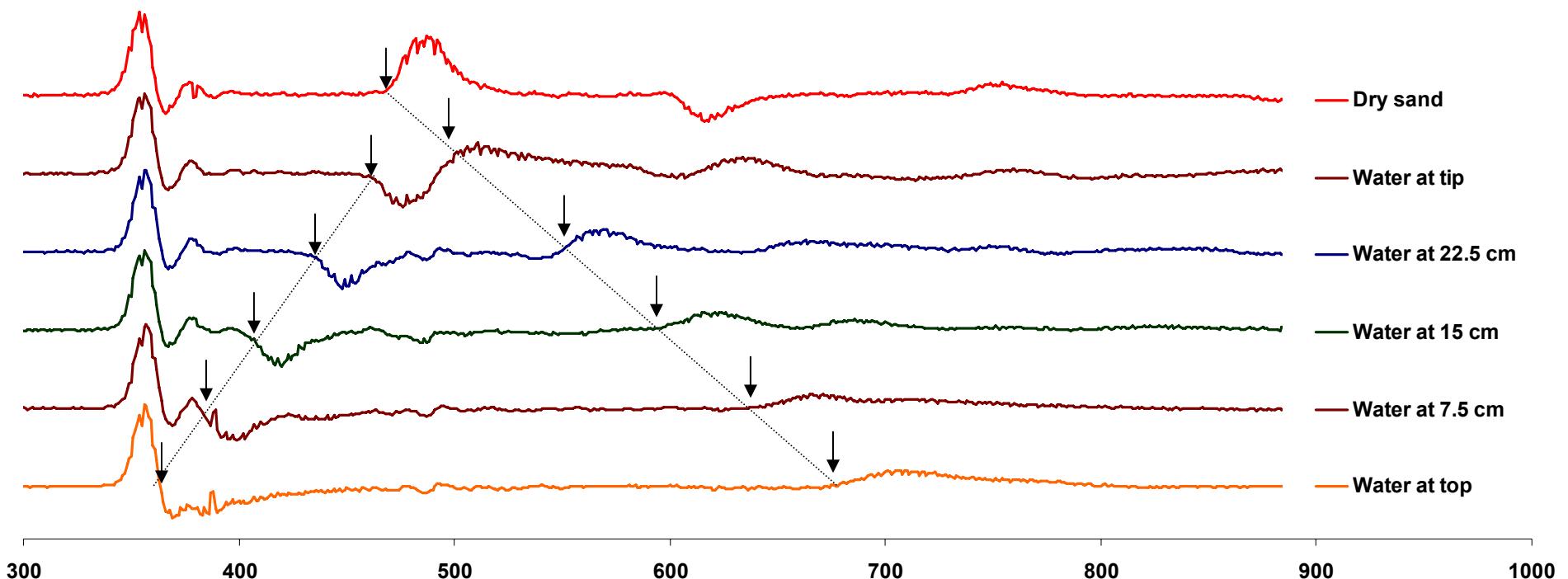
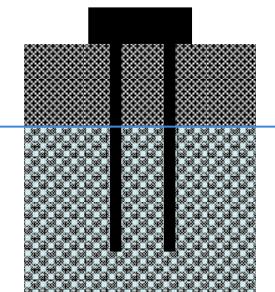
Cho, Lee

Heterogeneity - Layering



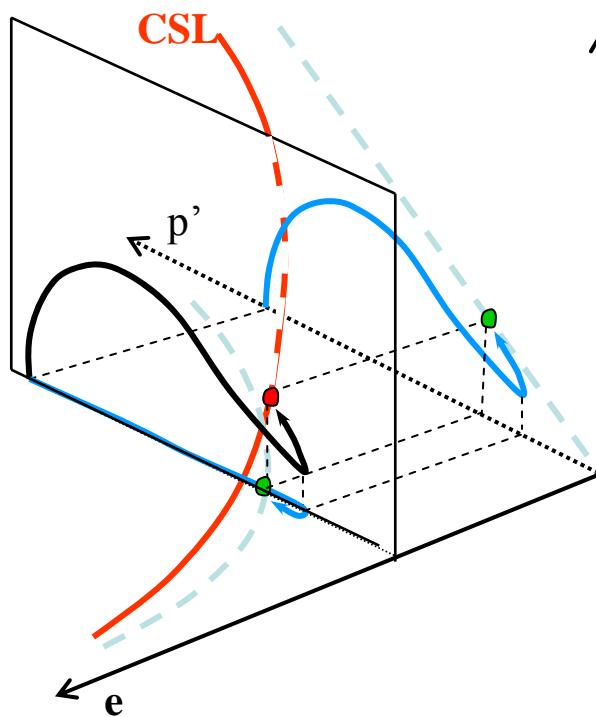
*a clay seam may hide the rest
(very high mismatch in this case)*

Heterogeneity in Water Content

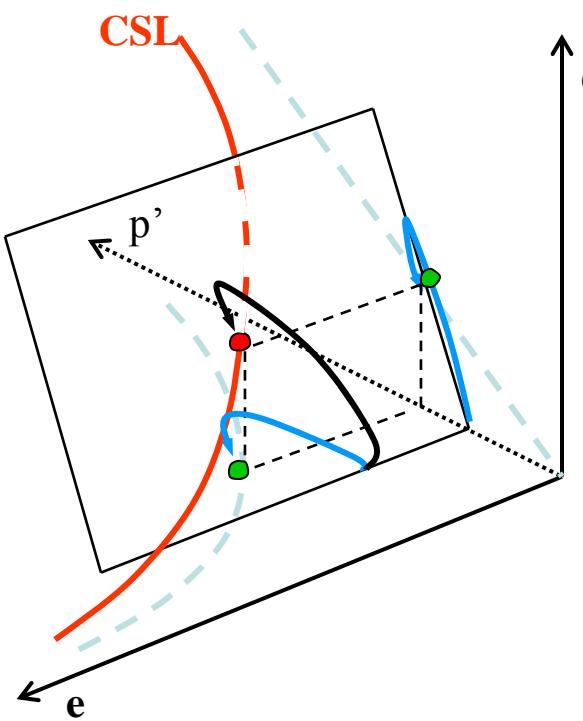


more than one primary reflections

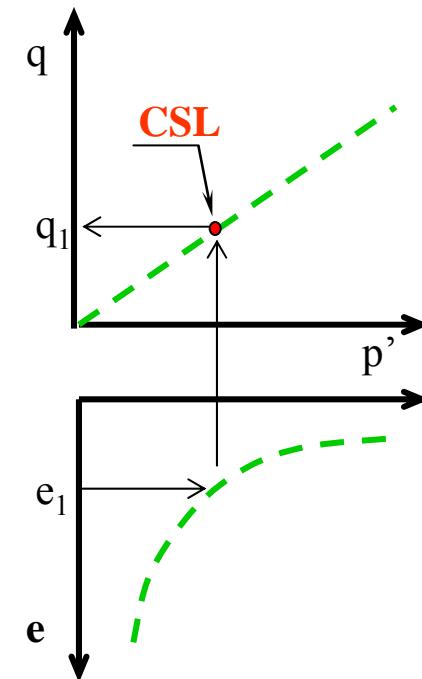
Insertion Effects



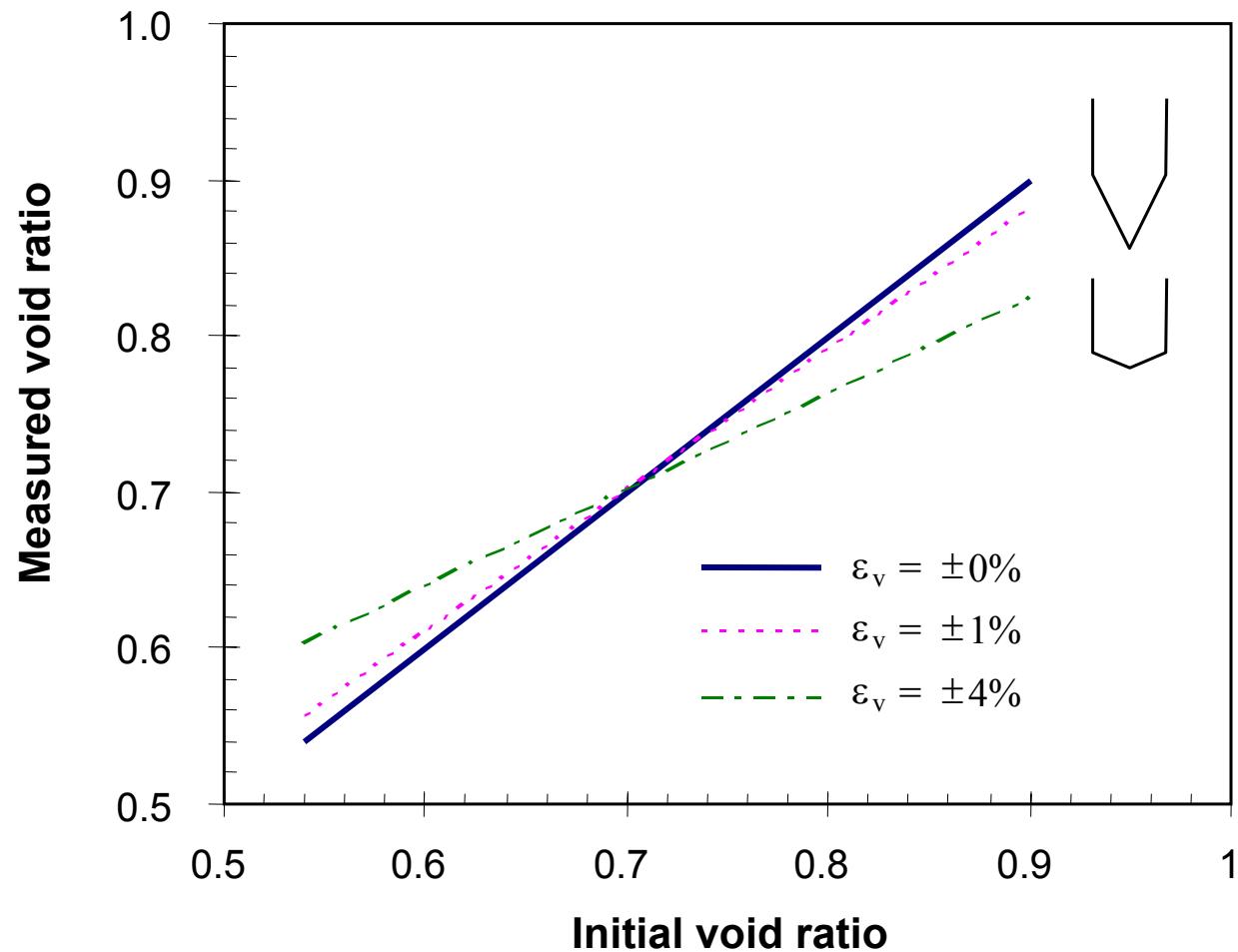
Undrained



Drained

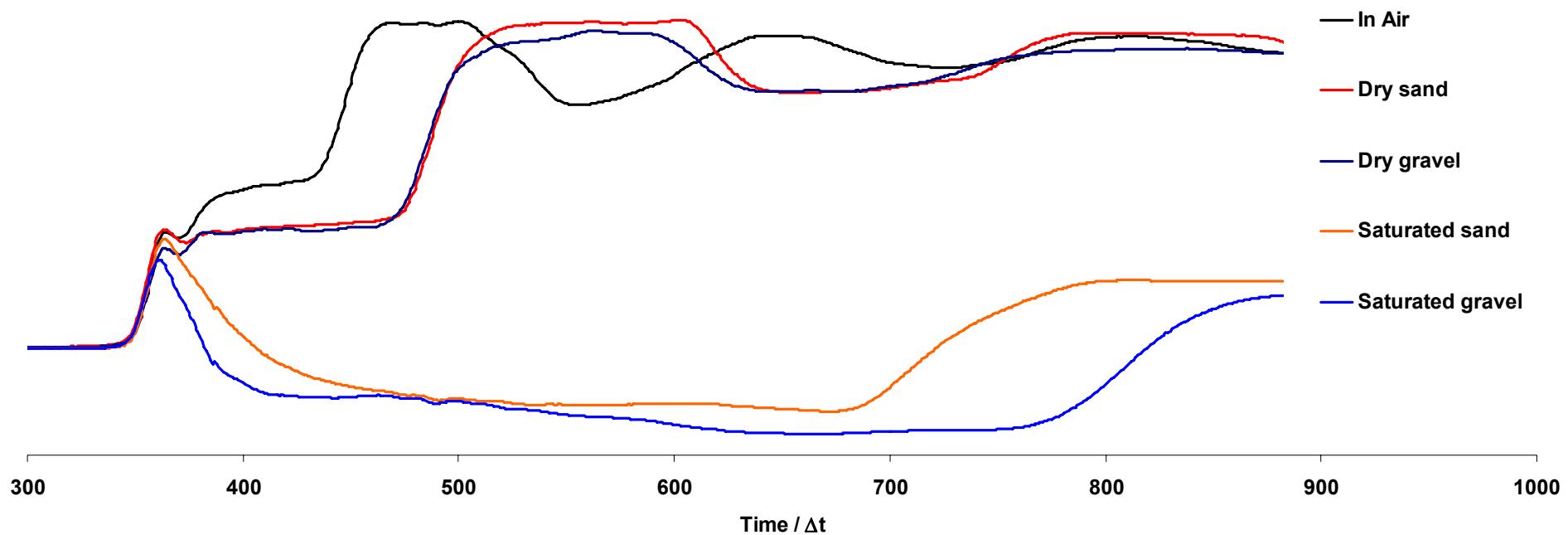


Insertion: Volumetric Strain = f(void ratio)



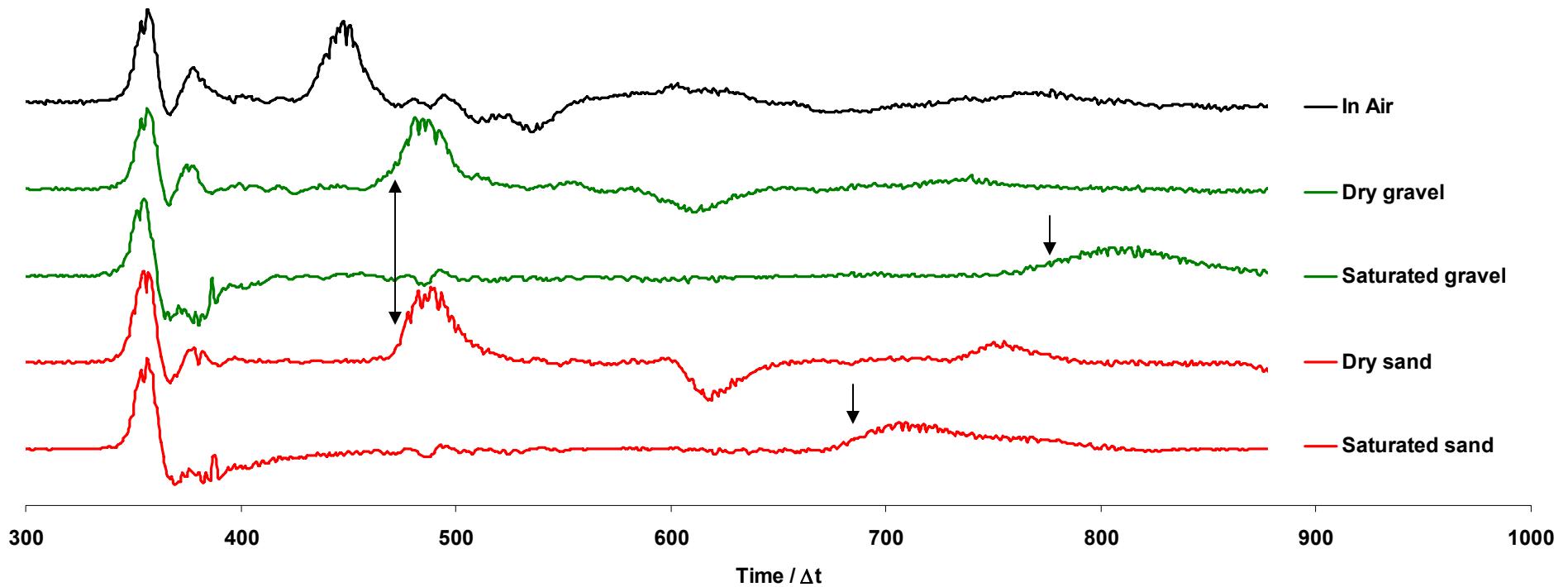
Large vs. Small Particles

Gravel – $d_{50}=20\text{ mm}$



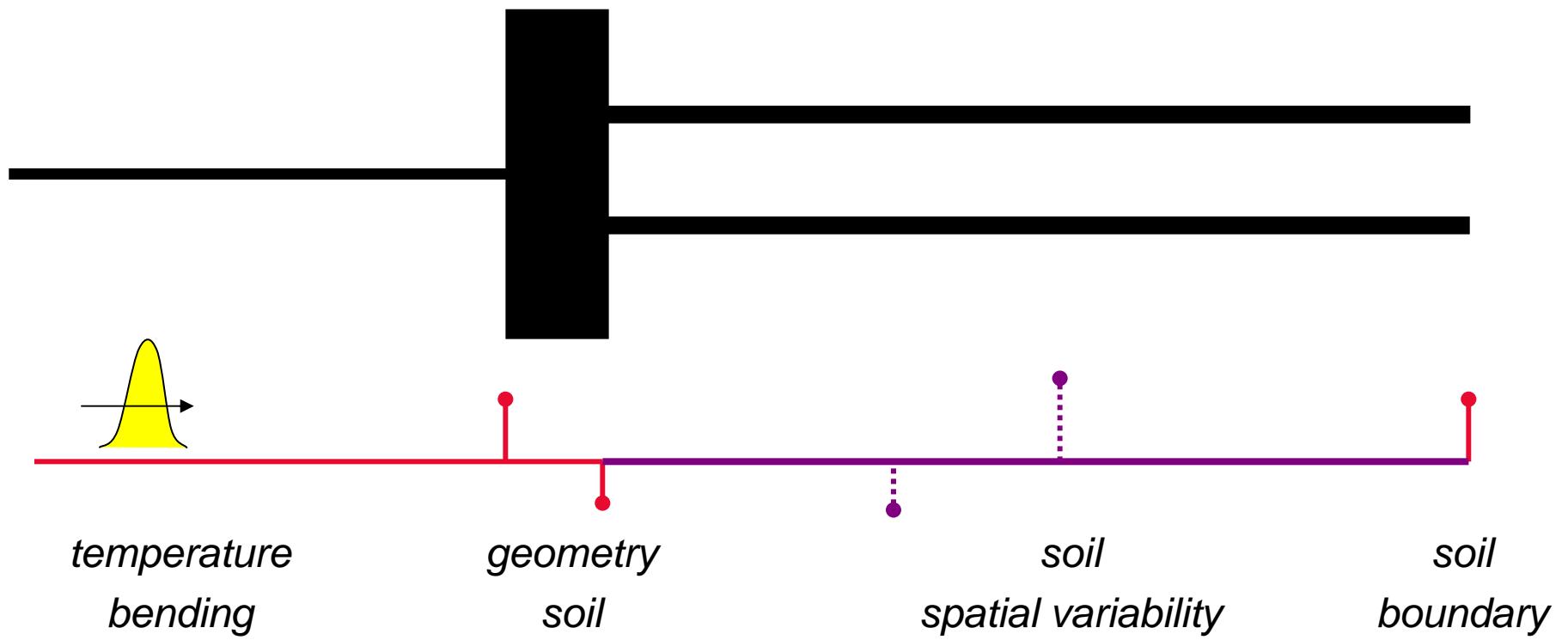
Large vs. Small Particles

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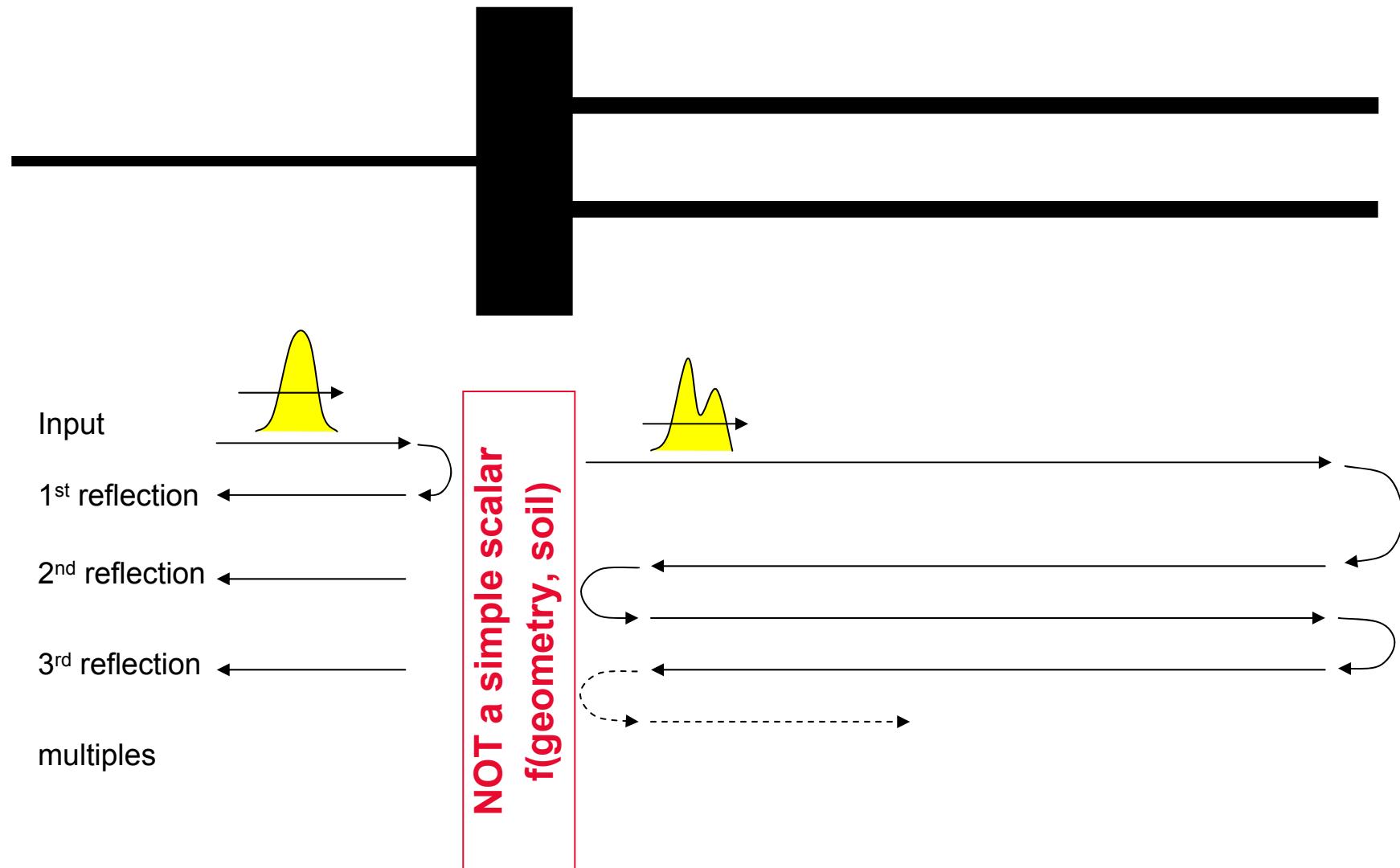


*higher local porosity in gravel
Brillouin LP filter?*

Summary



Summary



Summary

The connection to probe:

sequence of electrical and geometrical changes

response is a function of the soil itself

when is time zero? what signal gets to the soil?

Compare the 2nd and 3rd reflections (if 3rd is not lost in noise)

Geometric dispersion + attenuation: signal widens

Ferromagnetism: expect small effect

Insertion effects and preferential packing (aggravated in coarse soils)

Complex signal: consider spatial variability

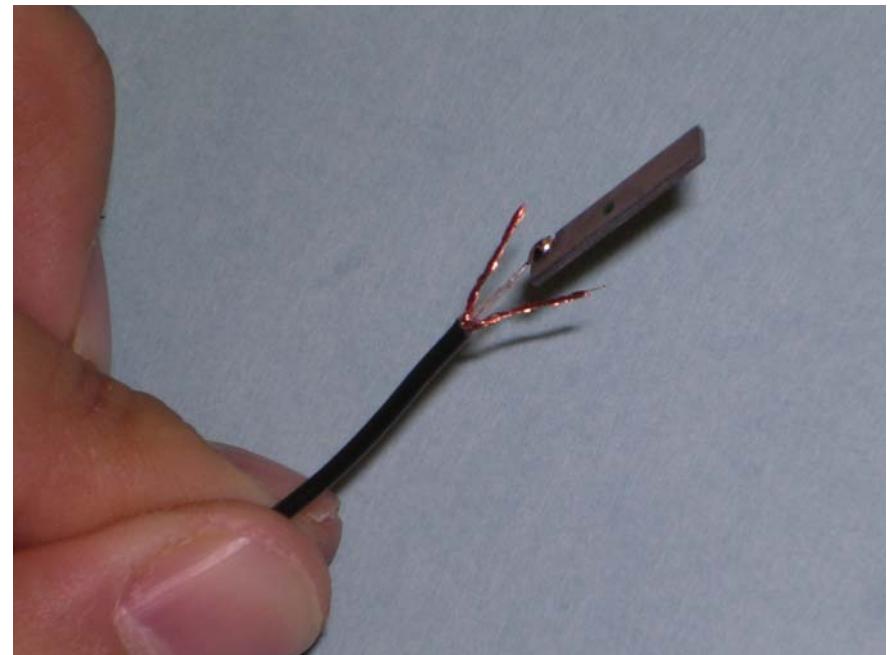
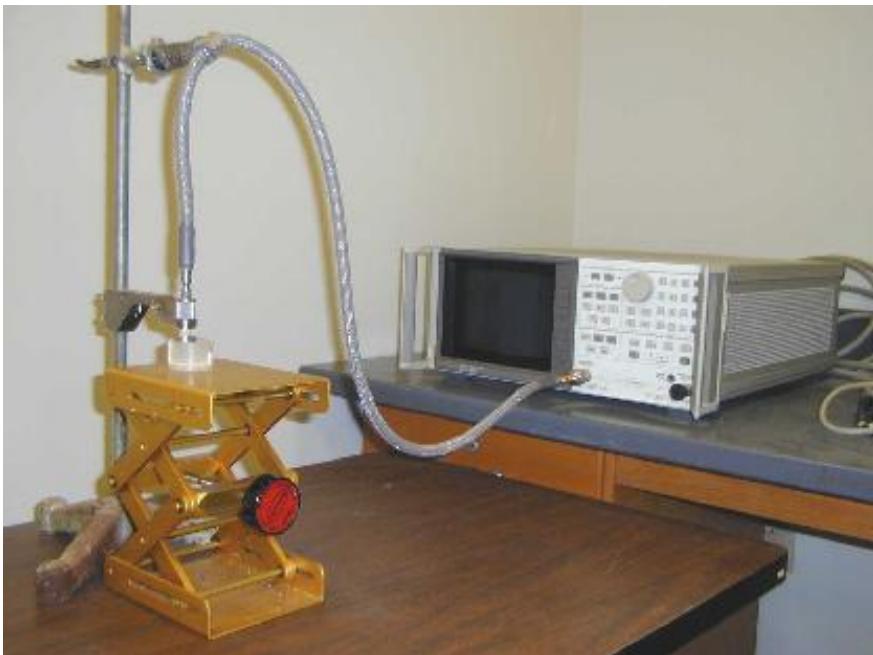
multiple interpretations of multiples

many unknowns → inversion may be ill defined

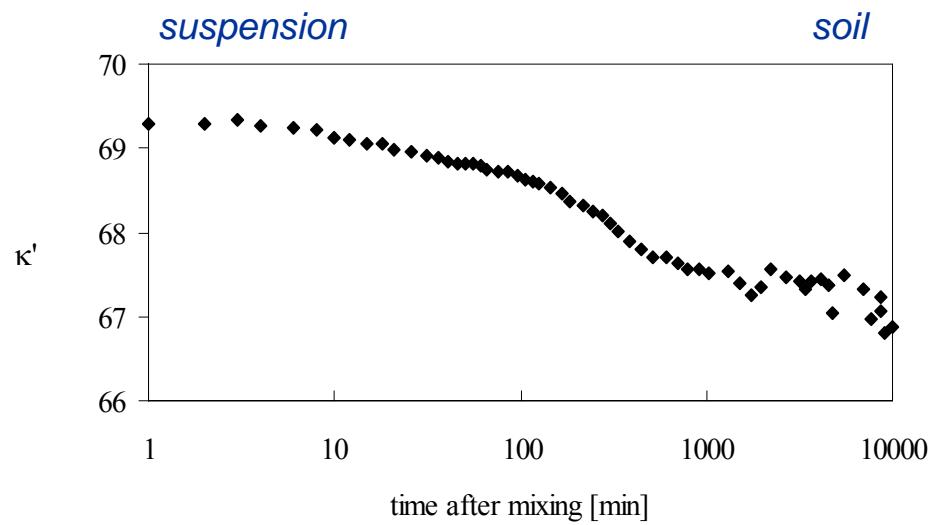
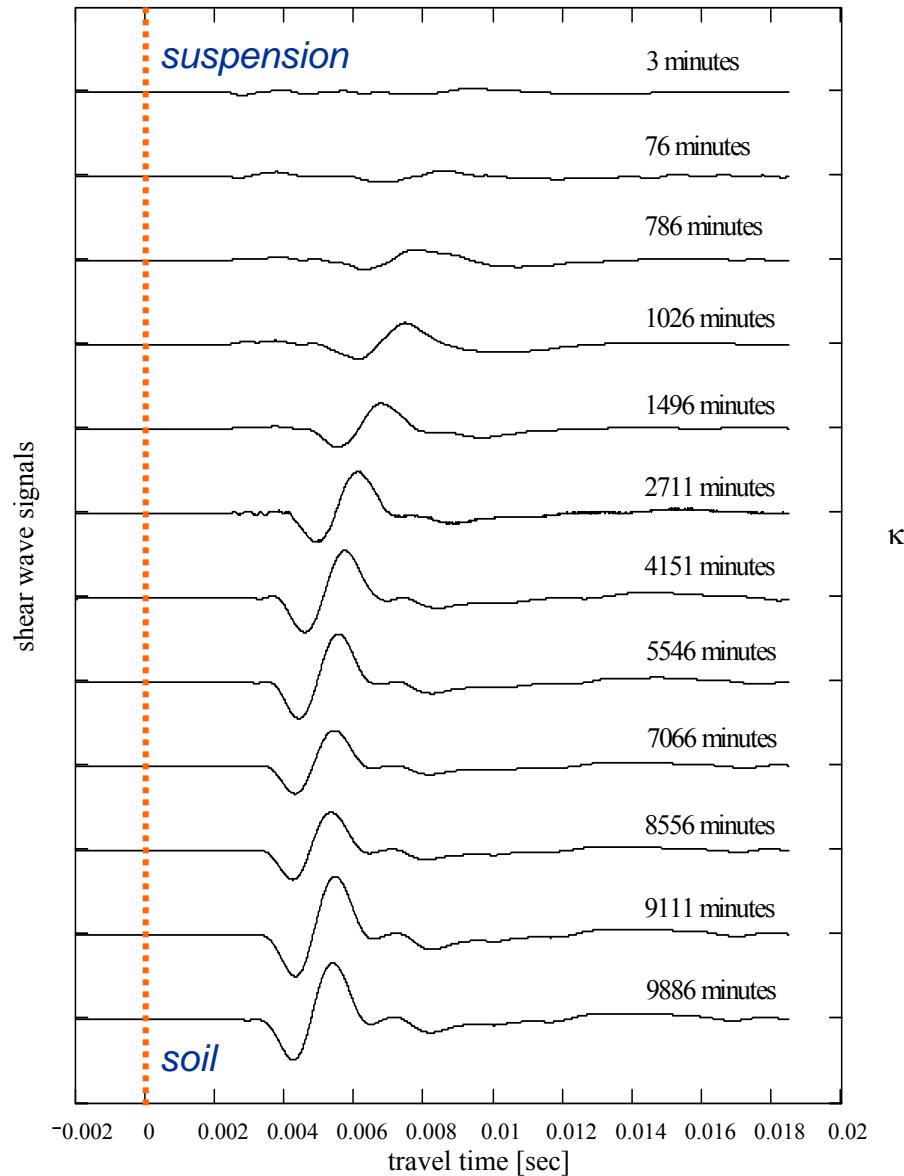
Information conservation → simple models (Ockham's criterion)

process monitoring

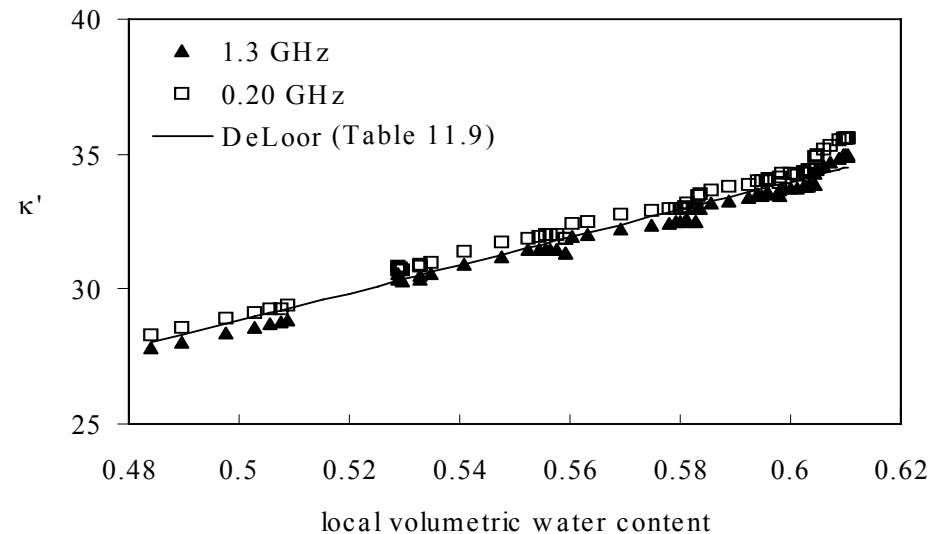
Measurements



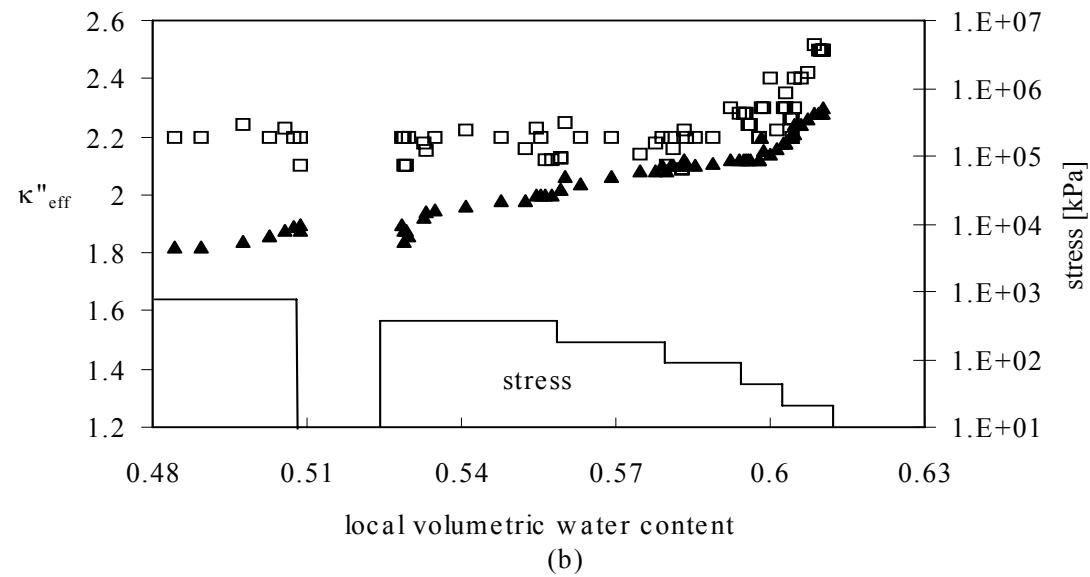
Sedimentation



Pressure diffusion

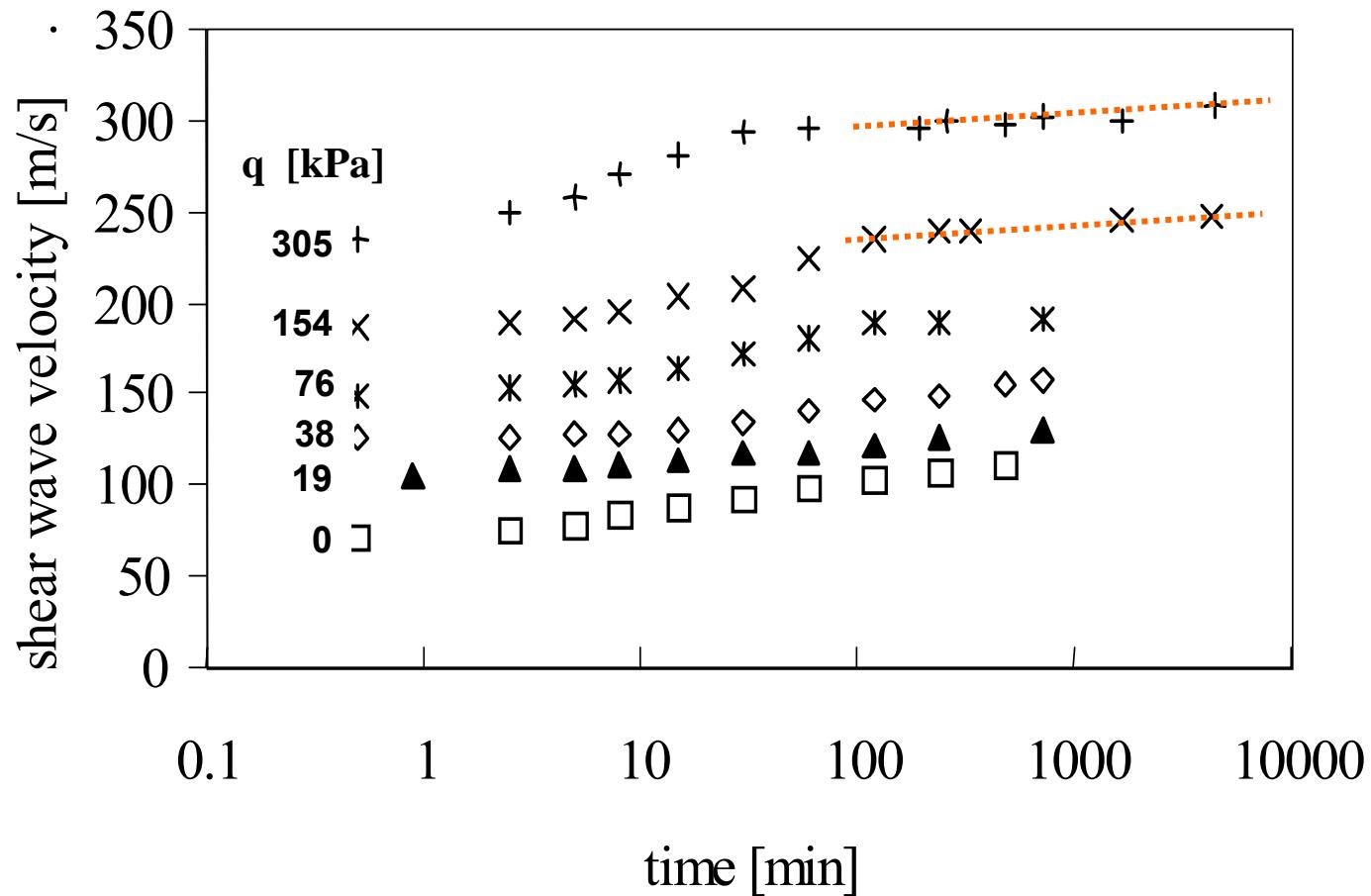


(a)



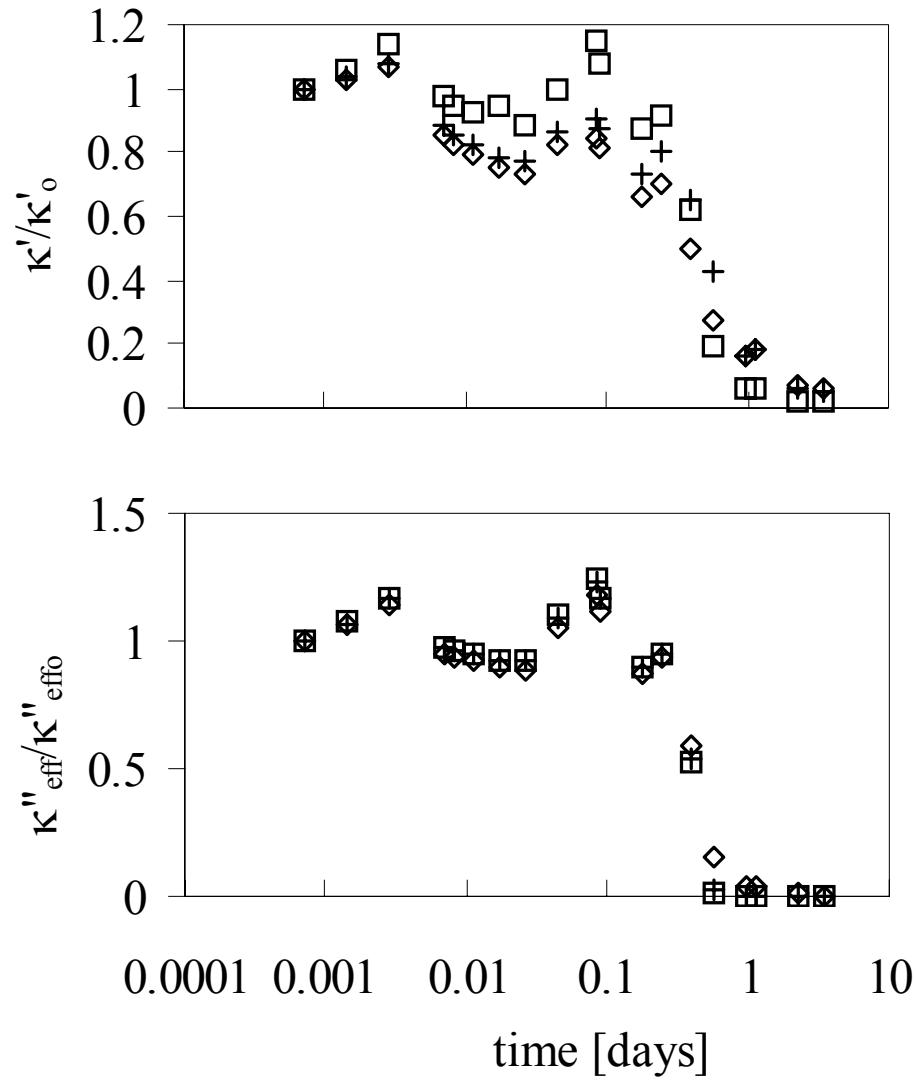
(b)

Pressure diffusion



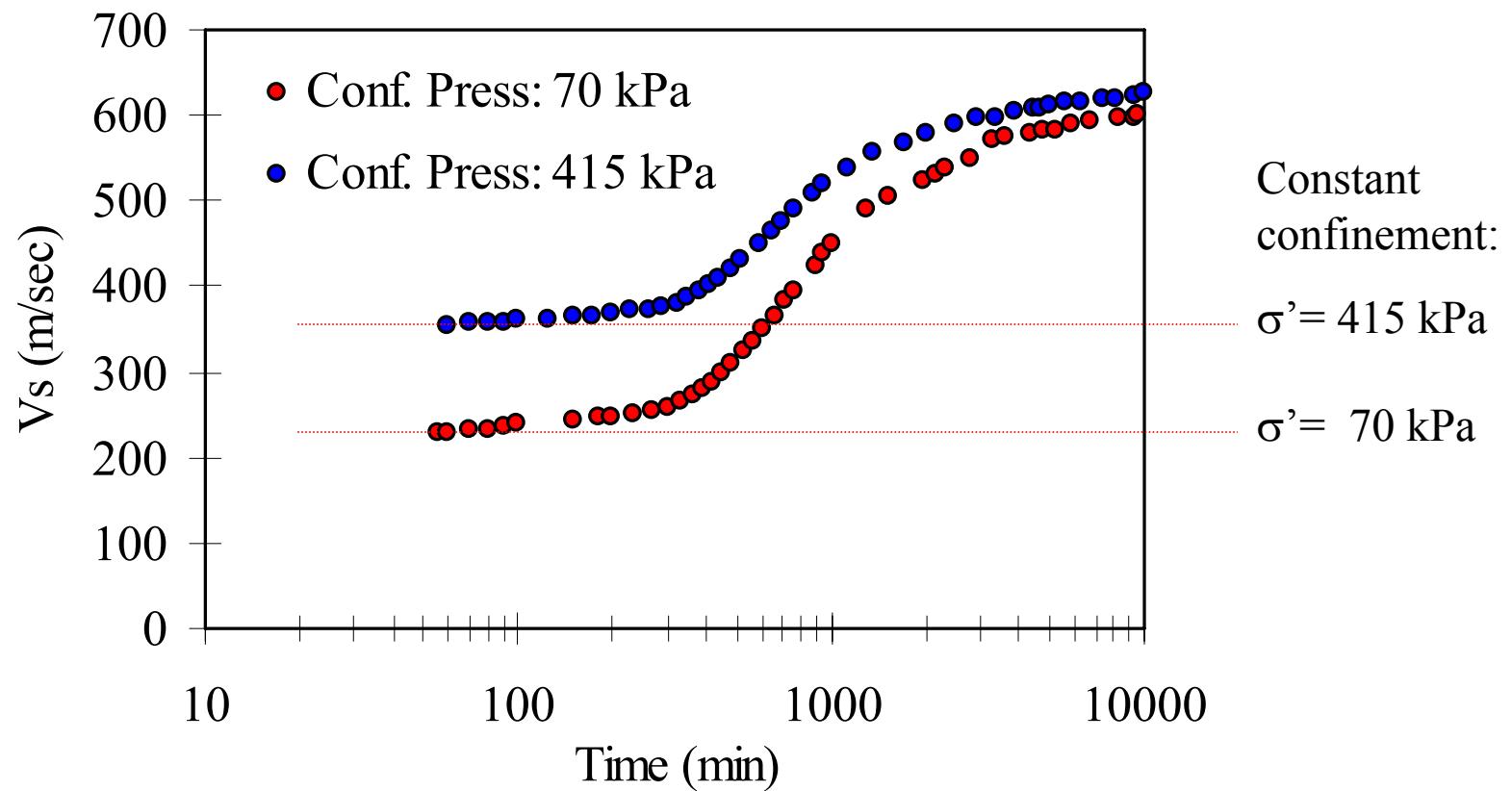
Cementation

(bentonite-cement)

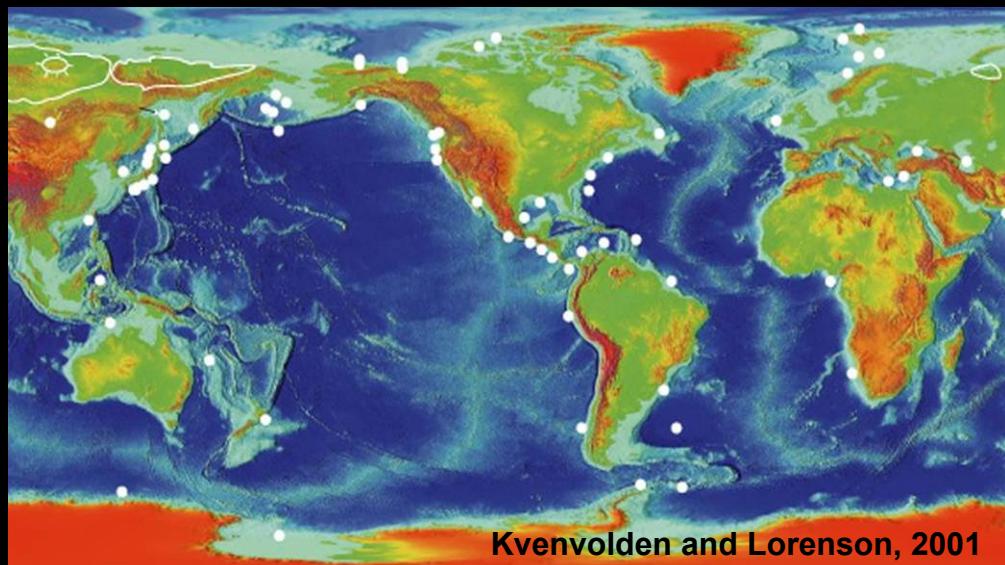


Cementation

(sand-cement)

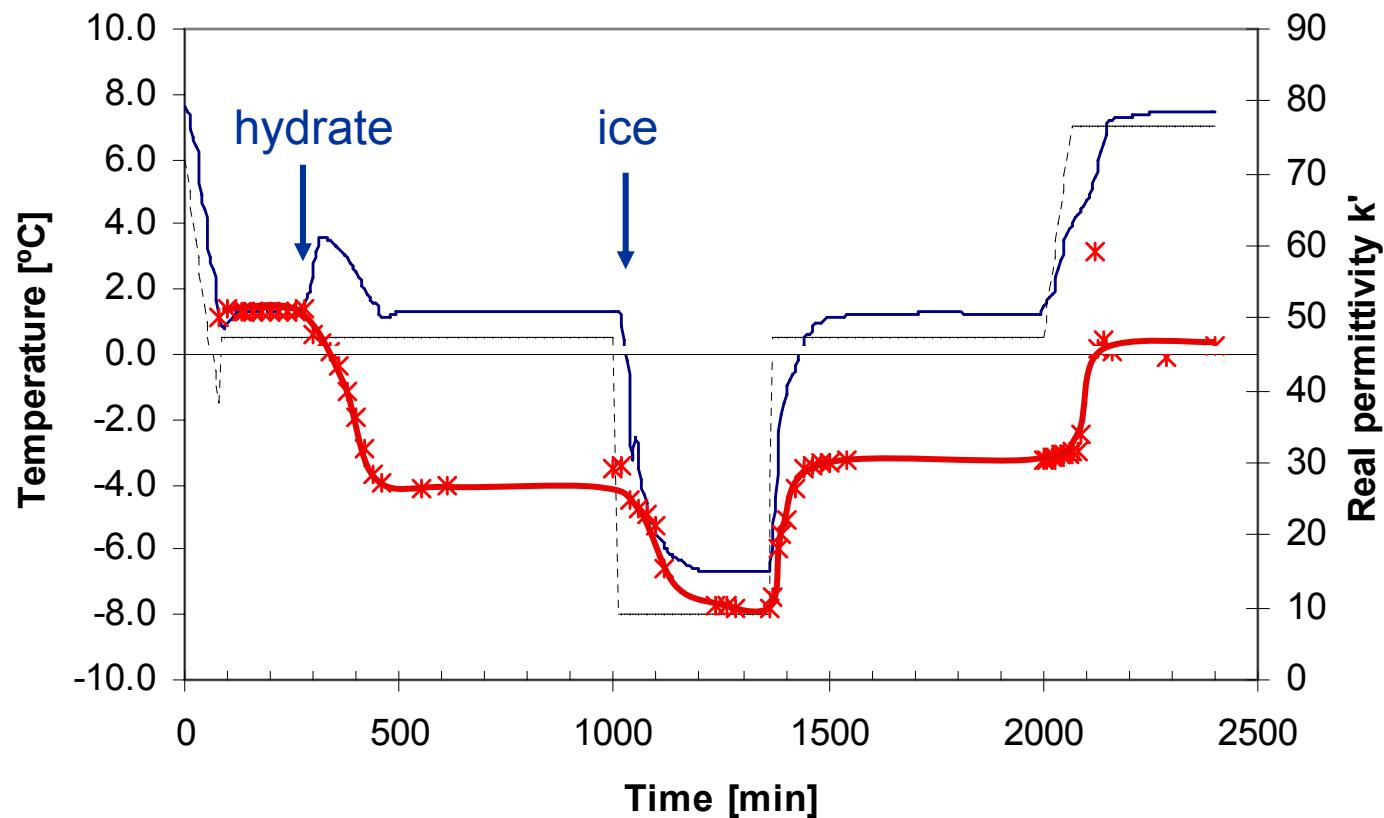


Gas hydrates

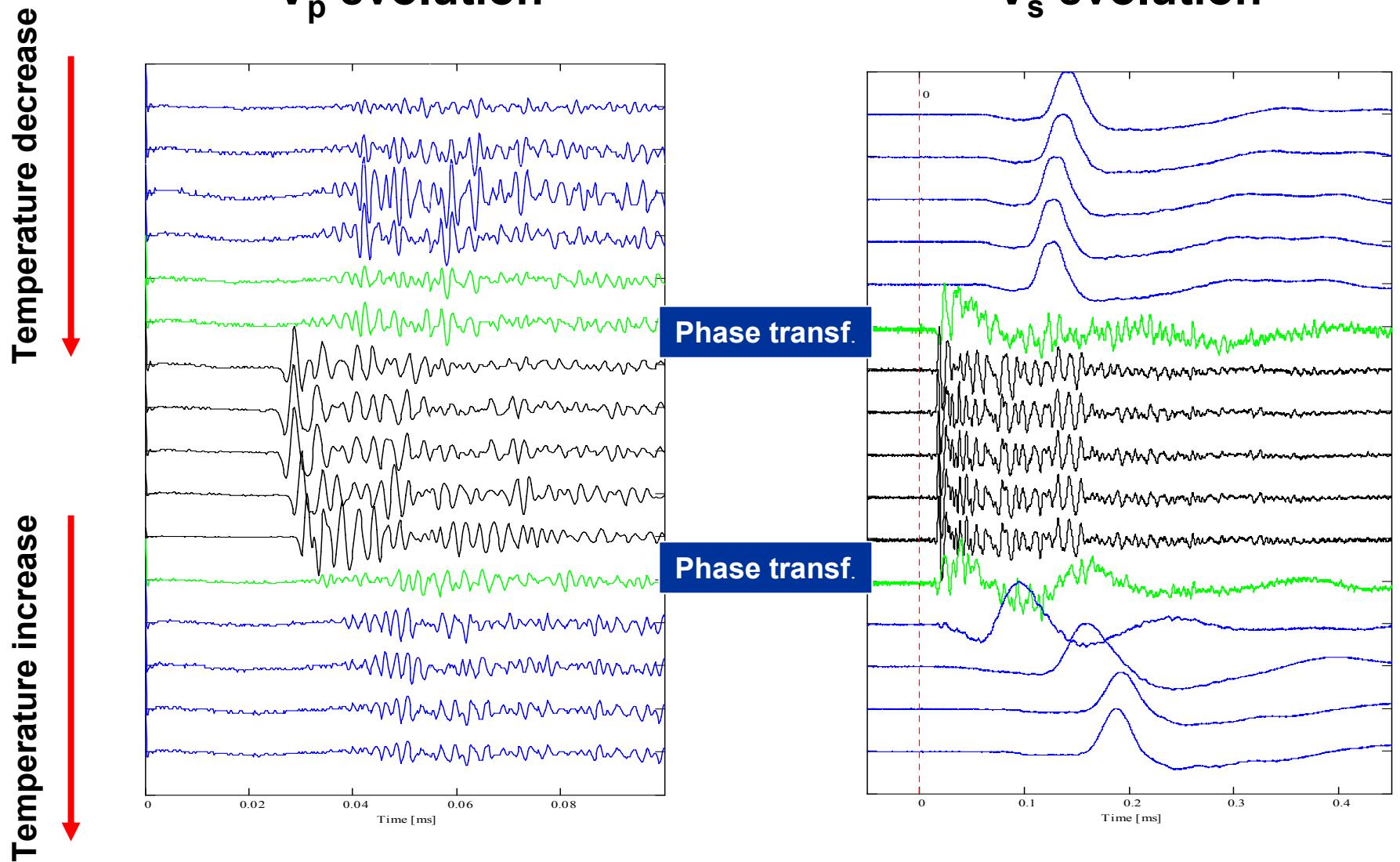


Real Permittivity

(Kaolinite + THF + H₂O)



Elastic waves



Penetration-based Geophysical Systems



SV Source
(Fernandez)

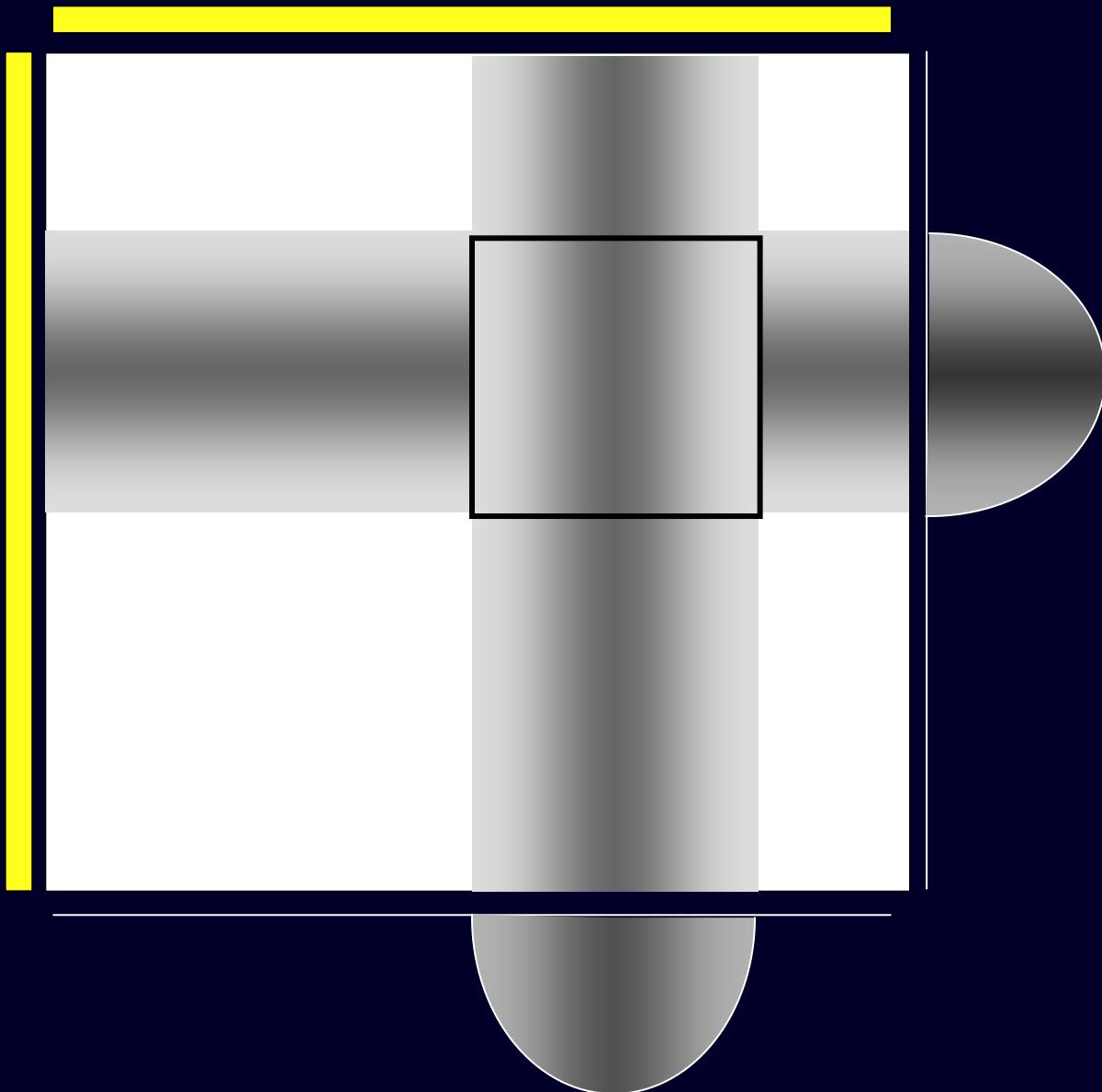


3D Geophone
(Stokoe – UT)



Conductivity tip

Boundary measurements - Tomography



closing thoughts

Measurement κ^* σ μ^*

TDR signature = input * (geometry AND spatially varying material)

Better measurement interpretation

Inversion: caution... follow Ockham's criterion

Inherent: insertion volume change

Consider non-intrusive implementation

Complementary information

Electromagnetic & elastic waves

Small perturbation & large-strain penetration testing

Wave parameters: relevant to engineering

Laboratory and field

Wide range of geotechnical processes

Boundary measurements: invert for internal conditions

Thank You

