J. Carlos Santamarina[1] *and Gye Chun Cho*[2]

Determination of Critical State Parameters in Sandy Soils—Simple Procedure

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ABSTRACT: The critical state is arguably the most robust criterion for strength design, including post liquefaction strength. The conventional triaxial test is used for the determination of critical state parameters; however, it is time-consuming and the required set of tests is relatively expensive for common geotechnical tasks. A simplified test procedure is developed to determine the critical state line in sandy soils. The procedure is reliable, economical, and fast. In order to verify the simplified test procedure, results are compared against critical state parameters determined in conventional triaxial tests. The comparison shows very good agreement between critical state parameters obtained with the suggested procedure and those gathered with triaxial testing. Limitations are identified.

KEYWORDS: critical state, index properties, Mohr-Coulomb failure criterion, sand, strength, triaxial testing

Coulomb in the eighteenth century understood that the strength of freshly remolded soils is of frictional nature, hence, stress dependent (Heyman 1997; Schofield 1998). Reynolds (1885) highlighted the tendency of granular materials to change volume when sheared, a fact that was well known by grain dealers at the time. Casagrande (1936) recognized that a critical density divides the tendency to volume change into contractive and dilative behaviors. Later, Taylor (1948) showed experimentally that dilatancy is stress dependent, and Bishop (1950) expressed the shear strength in terms of friction and dilatancy components. Finally, Roscoe, Schofield, and Wroth (1958) and Schofield and Wroth (1968) brought together stress-dependent strength and dilatancy in the unifying structure of critical state soil mechanics within the framework of plasticity theory.

Researchers in critical state (CS) soil behavior have generally relied on drained, strain-rate-controlled tests on dilatant specimens to determine the critical state because the critical state strength can be achieved at a relatively low global strain level (Been et al. 1991; Lee 1995). On the other hand, researchers in liquefaction have centered their efforts in undrained tests, usually on loose-contractive specimens to determine the steady-state line (Castro 1969; Poulos 1981; Poulos et al. 1985; Vaid and Chern 1985; Alarcon-Guzman et al. 1988; Konrad 1990; Ishihara 1993; Riemer and Seed 1997). However, dilative-drained tests and both contractive and dilative-undrained tests are prone to localization. Therefore, the measured global void ratio may deviate from the local void ratio in the shear band where large strains develop. Indeed, it appears that the best method to obtain critical state parameters would be to use homogeneous loose specimens subjected to drained shear; furthermore, such a test is not sensitive to incomplete saturation. However, a large strain level is needed.

The strain level required to achieve the critical state can challenge experimental design. While relatively low strains are needed to alter the network of interparticle forces, a micro-scale conceptualization of the problem suggests that strains in excess of 100% are needed so that particles have high probability of exchanging neighbors to attain the unique fabric conditions that correspond to critical state. Direct shear test data by Taylor (1948) obtained for Ottawa standard sand (specimen thickness $t = 10.4$ mm) show that the deformation required to reach critical state is about $\delta = 5.1$ mm. Thus, the average strain level required to reach critical state is $\gamma_{cs} \approx \delta/t \approx 50\%$. Furthermore, if strain localization is assumed in a region of thickness $t^* \approx 10\ D_{50}$, then the required level of "local" strain is greater than about 100%.

Such strain levels are not achievable in triaxial testing. Indeed, drained tests clearly show that critical state is not reached at the standard 20% strain limit. On the other hand, undrained tests suffer from complex poroelastic effects and localization.

Still, the conventional triaxial test is commonly used for determining the locus of critical states in the e-p'-q space. For convenience, the critical state line can be expressed in terms of its 2D projection on the p'-q space in terms of the strength parameter M and its 2D projection on the e-log p' space in terms of the critical state parameters slope λ and intercept Γ. Conceptually, three measurements are required to determine M, λ, and Γ. However, several tests are often run to compensate experimental variability. To some extent, the time-consuming and cumbersome task of running multiple triaxial tests has hindered the application of critical state soil mechanics. A simplified test procedure is proposed next.

Simple CS Test

The "simple CS test" is designed to determine M, λ, and Γ in sandy soils. The device and procedure are described next.

Experimental Apparatus

Various concepts, devices, and designs were tested including: concentric cylinders, rolling balloons (p' can not be accurately obtained), sliding plates, etc. The selected method presented herein is a very simplified form of axi-symmetric triaxial testing. It allows for the proper measurement of all needed parameters and requires

[1] Professor, Civil and Environmental Engineering, Georgia Institute of Technology, Atlanta, GA 30332. URL: http://www.ce.gatech.edu/~carlos/

[2] Graduate student, Civil and Environmental Engineering, Georgia Institute of Technology, Atlanta, GA 30332.

minimum, low-cost components that are readily available in all geotechnical laboratories and field installations.

Figure 1 shows the experimental setup for the simple CS test procedure. The following components are needed: vacuum, thin latex membrane, two plexiglass caps, two O-rings, a graduated cylinder, a porous stone, and a transparent hose. The plexiglass cap with the porous stone is connected to one end of the transparent hose, which is used to monitor the volume change in the specimen. The other end of the hose is connected to the vacuum system, which is used to apply effective confinement.

Test Procedure

Specimen preparation and test procedure are summarized next:

1. *Preliminary measurements.*
   - Obtain the internal cross-sectional area of the transparent tube (A_t). The transparent tube must be semi-rigid, so its internal cross section does not change with pressure. In this study, Teflon I.D. 0.46 cm is used (wall thickness = 0.09 cm; nominal pressure = 1700 kPa).
   - Determine the unit weight of water (γ_w) at the laboratory temperature.
   - Measure the volume of the device components without soil (V_d), by placing them into the graduated cylinder filled with water. Include the plexiglass caps, the latex membrane, O-rings, and a pre-determined length of the transparent hose,
   - Determine the specific gravity (G_s) of the soil.
2. Determine the critical state friction angle (ϕ_{cs}) by pouring soil in the graduated cylinder filled with water (a transparent rectangular container is preferred). Tilt it and bring it back slowly to the vertical position. Measure the angle of repose in the middle region of the slope (Fig. 2; Cornforth 1973; Bolton 1986; Schofield 1999). Saturation is used to avoid capillary effects (other implications are discussed later in the text).
3. Pre-mix the soil (about 300 g) with excess water (about 2 L) and de-air the mixture. De-airing can be done by the application of vacuum or by boiling.
4. Prepare the specimen under water to avoid air entrapment. Gradually spoon the soil into the membrane filled with wa-

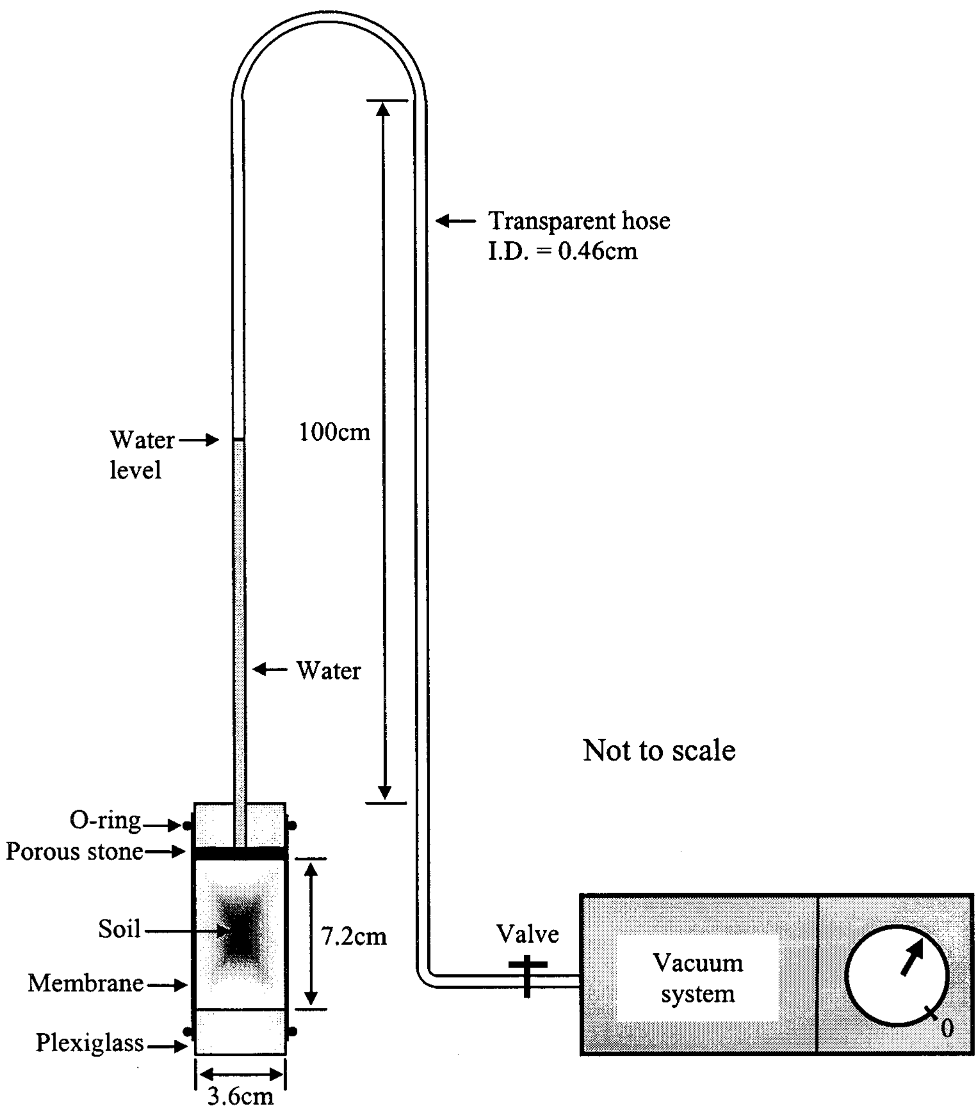

FIG. 1—*Device and experimental setup used for the simple CS test.*

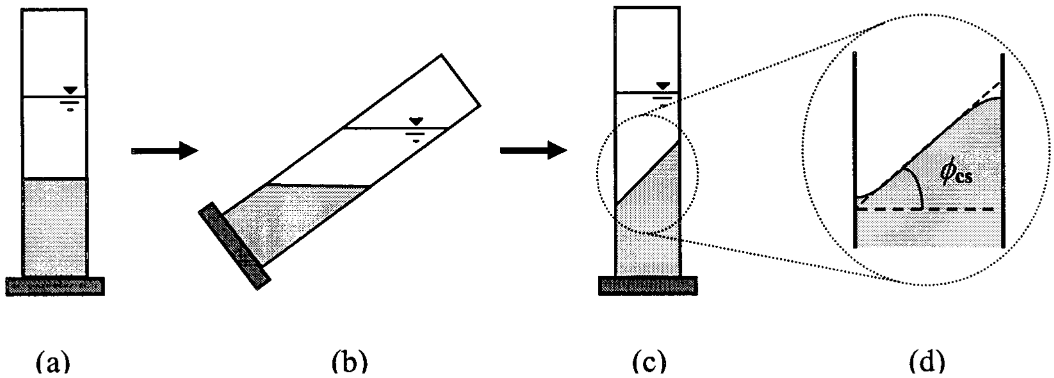

FIG. 2—*Simplified method to determine the critical state friction angle:* (a) *Pour soil in a 1000*-mL *cylinder with water.* (b) *Rotate the cylinder passed 60°.* (c) *Slowly return the cylinder to its vertical position and measure the angle of repose.* (d) *The angle is measured in the middle region of the slope.*

ter until the soil occupies a predetermined height about twice the diameter of the latex membrane. The transparent hose must be filled with water to a preselected height. Compaction is not needed because the soil is preferred in its loose state.

5. Place the top plexiglass cap by displacing the excess water and fasten the membrane to it with the O-ring. Prior to testing, apply maximum vacuum and knead the specimen to remove any entrapped air within the specimen and the transparent hose. Release the vacuum and reform the specimen to a loose state and in a cylindrical shape.
6. Subject the specimen to a low vacuum σ'_{c1}. Apply vertical loading by hand or clamp until the axial strain approaches about 40%. Register the elevation of the water h_1.
7. Release the vacuum. Let the membrane swell, loosen the soil by repeatedly turning over the specimen, and reform the specimen by hand to its original cylindrical shape. The specimen should be loose and homogeneous.
8. Apply a new pressure σ'_{c2}, and repeat Steps 6 through 7. Repeat these procedures at 5 ~ 6 different pressures while registering σ'_{ci} and h_i.
9. At any intermediate pressure, mark the water level (h_o), and measure the total volume (V_t) of soil as well as the device components using the graduated cylinder, as in Step 1.
10. Disassemble the device and determine the dry weight of the soil (W_s).
11. Determine the additional radial stress contributed by the membrane stiffness (σ'_m) by measuring the pressure that must be applied inside the membrane to cause it to expand laterally similar to the final expansion observed during the test (a value of $\sigma'_m = 7$ kPa is obtained for the standard membranes used in this study; at about 40% shortening, the maximum diameter the specimen reaches at middle height is 1.6 times of the initial diameter for a length-to-diameter ratio of 2).

Note that the deviator stress is not measured. While it is possible to measure it, it is not needed in this procedure, and it would add experimental complexity.

Data Reduction

The applied effective stress σ'_c is corrected for the stress σ'_m contributed by the membrane stiffness. The effective confining stress (σ'_3) becomes:

$$\sigma'_3 = \sigma'_c + \sigma'_m \tag{1}$$

For axisymmetric conditions, the mean principal stress p' and the deviator stress q are expressed in terms of the effective axial stress σ'_1 and the effective confining stress σ'_3,

$$p' = \frac{\sigma'_1 + 2\sigma'_3}{3} \tag{2}$$

$$q = \sigma'_1 - \sigma'_3 \tag{3}$$

For the soil at critical state, the effective axial stress σ'_1 is related to the effective confining stress σ'_3 and the critical state friction angle ϕ_{cs}:

$$\sigma'_1 = \sigma'_3\left(\frac{1 + \sin\phi_{cs}}{1 - \sin\phi_{cs}}\right) \tag{4}$$

By substituting the Eq 4 into Eqs 2 and 3, the mean principal stress p' and the deviator stress q at the critical state become:

$$p' = \sigma'_3\left(\frac{3 - \sin\phi_{cs}}{3(1 - \sin\phi_{cs})}\right) \tag{5}$$

$$q = \sigma'_3\left(\frac{2\sin\phi_{cs}}{1 - \sin\phi_{cs}}\right) \tag{6}$$

The strength parameter M is the ratio between q and p' at critical state. From Eqs 5 and 6, M in axial compression is:

$$M = \frac{q}{p'} = \frac{6\sin\phi_{cs}}{3 - \sin\phi_{cs}} \tag{7}$$

The specimen volume V_{sp} is calculated by subtracting the device volume V_d from the total volume V_t:

$$V_{sp} = V_t - V_d \tag{8}$$

The reference water volume V_{wo} is obtained by considering the soil volume V_s ($V_s = W_s/G_s\gamma_w$) and the measured specimen volume V_{sp}:

$$V_{wo} = V_{sp} - V_s \tag{9}$$

The volume change ΔV_i at the i-th pressure becomes:

$$\Delta V_i = d_i A_t \tag{10}$$

where d_i is the distance between h_o and h_i (i.e., $d_i = h_o - h_i$) and A_t is the inside cross-sectional area of the transparent hose. Therefore, the water volume V_{wi} at the i-th pressure is expressed as the reference water volume V_{wo} and the volume change ΔV_i:

$$V_{wi} = V_{wo} + \Delta V_i \tag{11}$$

Finally, the void ratio e_i at a given pressure p_i' is calculated from the initial volume and the measured volume changes in the transparent hose:

$$e_i = (V_{wi} G_s \gamma_w)/W_s \quad (12)$$

The calculation procedure is summarized in Table 1.

Values of λ and Γ are obtained by plotting the void ratio e versus the mean principal stress p' in logarithmic scale. The best-fit line is the projection of the critical state line on the e-log p' space. The value of λ is the slope of this line, and the intercept Γ is the void ratio at $p' = 1$ kPa.

Experimental Study—Verification

The proposed methodology is evaluated with eight soils. In addition, drained and undrained standard triaxial tests are also con-

TABLE 1—*Data reduction chart.*

Soil designation:	Area inside tube (A_t): Step 1
Volume of device without soil (V_d): Step 1	Unit weight of water (γ_w): Step 1
Additional radial stress (σ_m'): Step 11	Specific gravity (G_s): Step 1
Critical state friction angle (ϕ_{cs}): Step 2	Total volume (V_t): Step 9
Soil weight (W_s): Step 10	Specimen volume (V_{sp}): $V_{sp} = V_t - V_d$
Soil volume (V_s): $V_s = W_s / G_s \gamma_w$	Reference water volume (V_{wo}): $V_{wo} = V_{sp} - V_s$

#	Applied pressure (σ_c')	Correction (σ_3')	**Mean principal stress (p')**	Distance (d_i)	Volume change (ΔV)	Water volume (V_w)	**Void ratio (e)**
i	σ_{ci}'	$\sigma_{3i}' = \sigma_{ci}' + \sigma_m'$	$p_i' = \sigma_{3i}' \left(\frac{3 - \sin\phi_{cs}}{3(1 - \sin\phi_{cs})} \right)$	$d_i = h_o - h_i$	$\Delta V_i = d_i A_t$	$V_{wi} = V_{wo} + \Delta V_i$	$e_i = (V_{wi} G_s \gamma_w)/W_s$

Plot the values of e-vs-log p' to determine slope λ and intercept Γ.

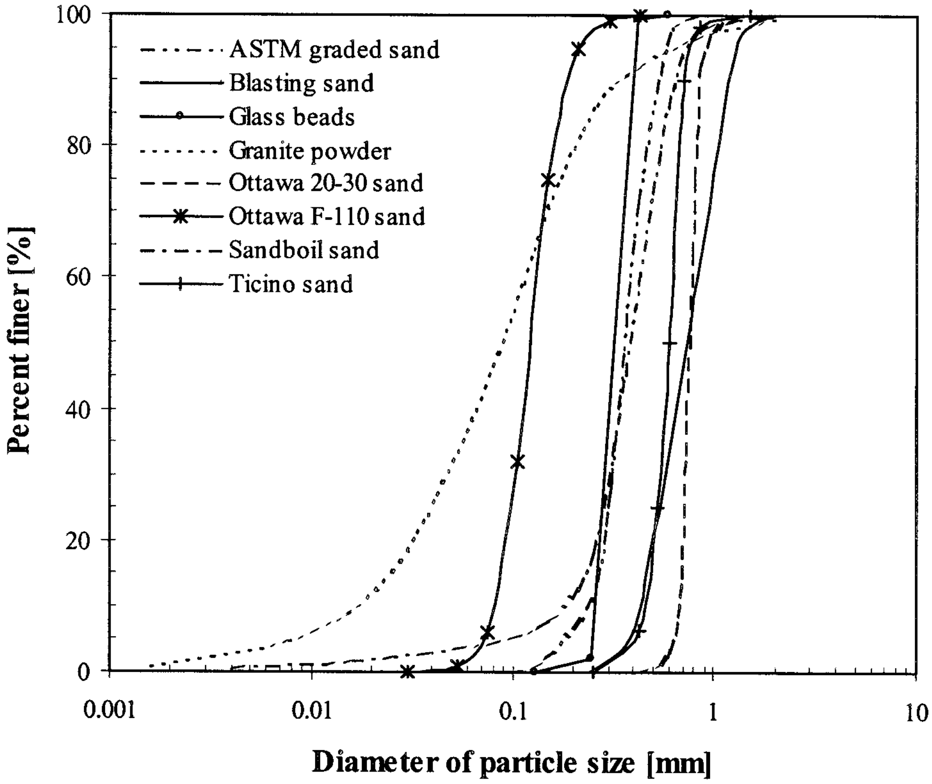

FIG. 3—*Particle-size distribution for the tested soils.*

TABLE 2—*Tested materials—properties.*

Material	e_{max}	e_{min}	D_{50} (mm)	D_{10} (mm)	C_u	C_c	G_s
ASTM graded sand	0.820	0.500	0.35	0.23	1.65	1.06	2.65
Blasting sand	1.025	0.698	0.71	0.42	1.94	0.94	2.65
Glass beads	0.720	0.542	0.32	0.24	1.37	0.99	2.46
Granite powder	1.296	0.482	0.089	0.017	6.18	1.08	2.75
Ottawa 20–30	0.742	0.502	0.72	0.65	1.15	1.02	2.65
Ottawa F-110 sand	0.848	0.535	0.12	0.081	1.62	0.99	2.65
Sandboil sand	0.790	0.510	0.36	0.17	2.41	1.29	2.62
Ticino sand	0.937	0.574	0.58	0.44	1.38	1.00	2.68

NOTE: C_u is the coefficient of uniformity ($C_u = D_{60}/D_{10}$), C_c is the coefficient of curvature ($C_c = D_{30}^2/D_{10} \cdot D_{60}$), and G_s is the specific gravity.

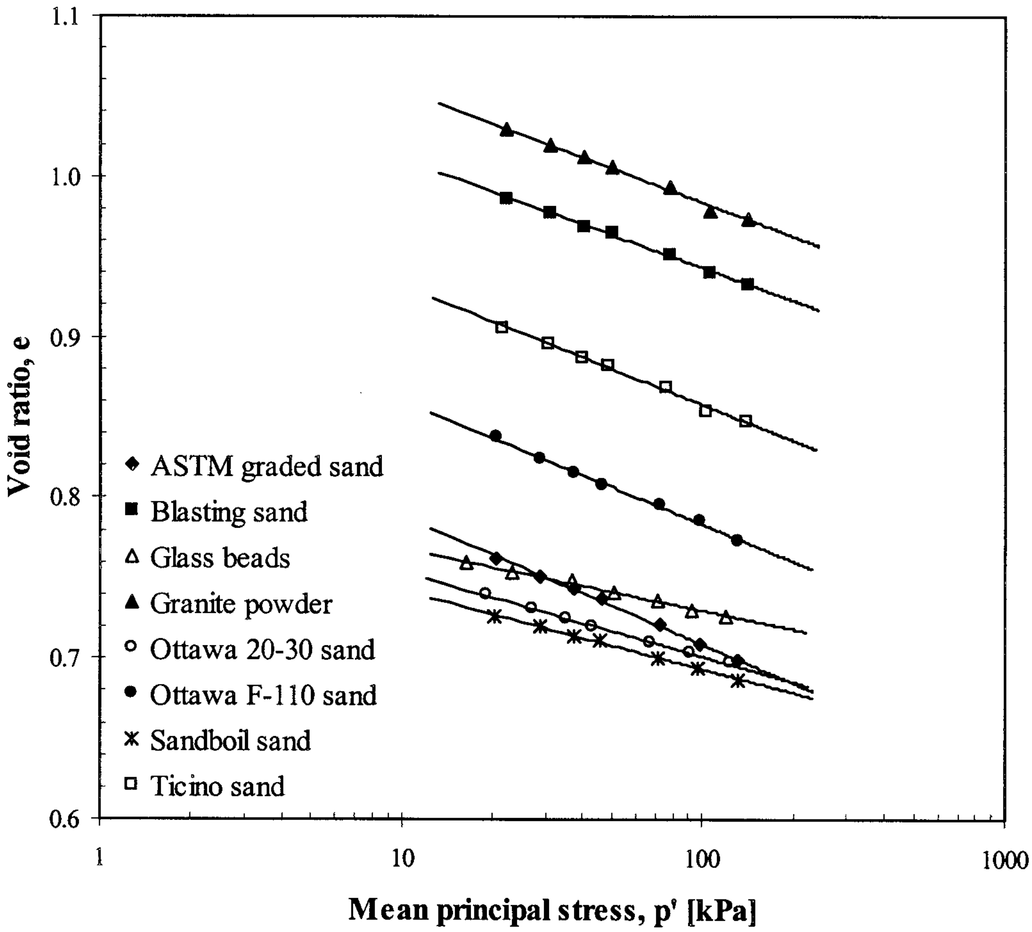

FIG. 4—*Simple CS test results—e-log* p′ *projection of the critical state line for the different soils.*

ducted to determine critical state parameters for three of the tested soils.

Selected Soils

Eight soils are selected to run the simple CS test: ASTM graded sand, blasting, glass beads, granite powder, Ottawa 20–30 sand, Ottawa F-110 sand, sandboil sand, and Ticino sand. Figure 3 shows the particle size distribution, and Table 2 summarizes the main properties for these soils. ASTM graded, Ottawa 20–30, and Ottawa F-110 sands are subround, while blasting sand and Ticino sand are angular. Granite powder is a byproduct of rock crushing during the production of aggregates for concrete, and it is a well-graded material composed of angular grains. The sandboil sand is a natural soil from a paleoliquefaction site in mid-America and has some fine content. Glass beads are studied because they provide an extreme case for uniformity, roundness, and sphericity.

Test Results

Figure 4 shows the results obtained with the simple CS test in *e*-log p' space for the eight different soils. The critical state friction angle ϕ_{cs}, the slope λ, and the intercept Γ of critical state line are summarized in Table 3.

Verification

In order to verify the simple CS test for the determination of critical state parameters, conventional triaxial tests are performed for Ottawa 20–30 sand, blasting sand, and sandboil sand. Loose ho-

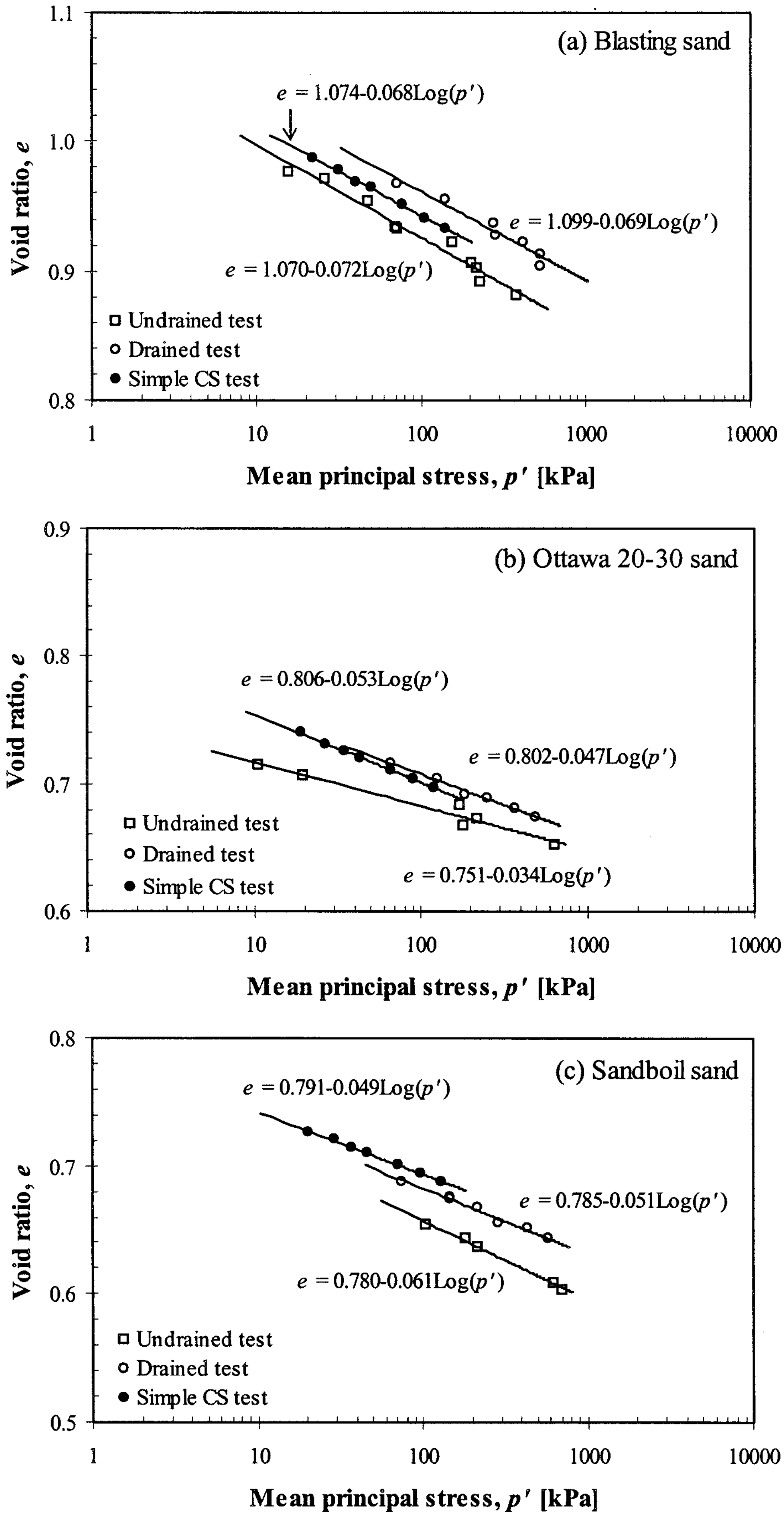

FIG. 5—*Comparison between simple CS test results and triaxial test results.*

TABLE 3—*Simple CS test results—Critical state parameters for several soils.*

Material	Friction Angle, ϕ_{cs}	Intercept of CSL at 1 kPa in e-log p'	Slope of CSL in e-log p'
ASTM graded sand	30°	0.869	0.080
Blasting sand	34° (32°)	1.074 (1.099)	0.068 (0.069)
Glass beads	21°	0.807	0.039
Granite powder	34°	1.124	0.070
Ottawa 20–30	28° (27°)	0.806 (0.802)	0.053 (0.047)
Ottawa F-110 sand	31°	0.937	0.077
Sandboil sand	33° (33°)	0.791 (0.785)	0.049 (0.051)
Ticino sand	33° (34°*)	1.006 (0.946*)	0.074 (0.04*)

NOTE: Values in parentheses correspond to drained triaxial test results.
* The values in parentheses for Ticino sand were obtained with undrained triaxial tests (provided by D. Lo Presti).

mogeneous specimens and drained tests are used to avoid strain localization. Results are summarized in Table 3 and shown in Fig. 5. For comparison, undrained test results are also shown in Fig. 5. There is good agreement between the simple CS test results and the drained triaxial test results. It is important to note that the simple CS test data are always closer to the critical state line obtained with drained loose homogeneous specimens than results obtained with the more standard approach based on the undrained loading of loose specimens.

Discussion and Final Remarks

The friction angle is checked for repeatability (the intercept Γ and slope λ are determined from well-correlated multiple data points). Seven engineers performed three measurements of friction angle for blasting sand (angular), granite powder (very fine), and glass beads (round). The average friction angle is the one reported in Table 3; the standard deviation is less than 1.5°, so that the coefficient of variation is less than 5%. A $\pm 1.5°$ error in the friction angle results in $\pm 3.5\%$ error in the computed mean principal stress at critical state, p'_{cs}; this error affects only the computation of the intercept Γ.

The measurement of the friction angle under water is recommended to avoid interparticle capillary forces. However, silty sands or soils with a fine fraction like the granite powder or sandboil sand specimens may liquefy during the measurement of the friction angle, reducing a value lower than ϕ_{cs}. In this case, it is recommended to ovendry the soil and follow the simplified procedure without water.

The most critical measurement in the simple CS test is the determination of the specimen volume V_{sp}. Still, as compared to triaxial testing, the measurement of void ratio in the simple CS test is reliable and easy. The experience gathered during this study shows that the proposed procedure renders repeatable and consistent results. The methodology can be augmented by correlations between critical state parameters and index properties such as e_{max}, e_{min}, C_u, D_{50}, particle roundness, and sphericity (Cho and Santamarina 2001).

The presence of fines adds other difficulties related to segregation and the development of interparticle electrical forces. This occurs primarily with submicron particles, often of clay minerals. The validity of the methodology for sands with fines is questionable.

End restrain effects near the specimen cap and base cause nonhomogeneous deformation and may lead to a non-representative void ratio. However, Desrues et al. (1996) conclude that for loose drained specimens, there is no effect of localization so that the specimen experiences homogeneous deformation up to 40% axial strain.

When bulk water is subjected to vacuum, it begins to boil at room temperature when vacuum approaches 1 atm (101 kPa). However, dissolved air in water is released as bubbles at lower vacuum pressure. Air bubbles appear in boiled water and water deaired by DeAirator™ (Walter Nold Co.) when vacuum reaches ~90 kPa. Tap water releases air bubbles when the vacuum pressure is as low as 50 kPa. The simplified procedure, as proposed, uses boiled water and vacuum to confine the soil; hence, parameters are relevant to $\sigma'_c <$ ~90 kPa. The slope of critical state line in the e-log p' space may change at higher stresses due to particle breakage and other particle level processes (Been et al. 1991; Riemer and Seed 1997).

Membrane penetration effects are expected to be low given the low confining levels that are used. However, if the mean particle size D_{50} is greater than 0.2 mm, correction for membrane penetration may be required. This can be achieved by calibration prior to testing (Kramer and Sivaneswaran 1990).

Table 4 summarizes advantages and disadvantages for drained and undrained triaxial tests, and for the proposed simple CS test.

TABLE 4—*Advantages and disadvantages of different methods for the determination of critical state line.*

Test Method	Advantages	Disadvantages
Drained triaxial test (Apply deviator stress and measure volume change)	Any state of stress. Stress history modeling. Measures deviator stress. Can apply high confinement and neglect correction for membrane stiffness.	Time consuming and expensive. May need to correct for membrane penetration. Requires assembly of multiple specimens. Standard strain limitation ~20%. May be affected by localization.
Undrained triaxial test (Apply deviator stress and measure pore pressure)	Any initial state of stress. Stress history. Back pressure saturation (for proper pore-water pressure measurement).	Time consuming and expensive. May need to correct for membrane stiffness and penetration. Requires assembly of multiple specimens. High Skempton $B \approx 1$ is required. May be affected by localization.
Simple CS test (Measure volume change)	Easy procedure. Requires minimum equipment (no load cell). Same specimen is effectively reused. Takes few minutes to run a complete study. Specimens that experience localization are readily disregarded.	Requires well-saturated specimen and system. Cannot apply back pressure. Needs correction for membrane stiffness—affects Γ and λ. Critical state parameters apply to effective confinement $<$ ~90kPa.

The drained triaxial test performed on homogeneous loose specimens is the best standard method to obtain the critical state line, yet it is restricted by the strain level and cost. The suggested simple CS test device and procedure permit determining critical state parameters in minutes, reliably and with minimum cost equipment (stain level remains a limitation).

On the grounds of economics, effectiveness, and accuracy, the simple CS test procedure appears as a very convenient and reliable alternative approach for the determination of critical state parameters. While verification studies with soils from different regions are needed to corroborate the methodology, it can be concluded that the simple CS test procedure provides adequate critical state parameter, or (at least) good index values to predict the critical state soil parameters.

Acknowledgment

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