Modeling Bridge Deterioration with Markov Chains

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(Reviewed by the Highway Division)

Abstract: This paper describes methods for determining and utilizing Markov chains in the evaluation of highway bridge deterioration. Using a data base of 850 bridges in New York State, Markovian transition matrices (MTM) are first found for the overall bridge condition. Then, transition matrices are developed for the condition rating of individual bridge components (e.g., superstructures, decks, and piers). In each case, chains are determined for various types of construction. Also discussed is the modeling of correlated elements such as the primary structure and joint condition and the ability to determine the correlation for a set of data. The consequence of small data bases is discussed, and an explanation is offered for unexpected values of the transition probabilities. Finally examined is the use of Markovian analysis for predicting the evolution of the average condition rating of a set of bridges, and expected value of condition rating for a single bridge. Markov transition matrices are introduced to model the effects of repairs and to determine repair policies that will lead to constant average condition rating.

Introduction

All highway bridges are inspected at least every two years in accordance with federal standards. The inspector rates major structural and nonstructural elements, and provides "condition ratings" for each element, and a "general recommendation" for the bridge as a whole. The rating consists of mapping the assessed condition of a given component onto an (n, m) range, where n and m are integers. For example, the scale adopted by New York State Department of Transportation (DOT) uses the 1–7 range; the corresponding verbal definitions are shown in Table 1 (*Bridge Inspection* 1982; Bridge Inventory 1982).

The aging of bridges, as determined by means of evaluations at discrete times using an integer scale, can be readily modeled with a probabilistic approach as a discrete state Markov process. A Markov chain is a stochastic process in which the probability distribution in the next year depends only on the present distribution. For this study, each condition rating corresponds directly to a state in the Markov chain. When the transition probabilities are constant with time, the Markovian transition matrices (MTM) remain the same and the process is said to be stationary.

Previous work in Markovian analysis of highway bridges includes a study of transition probabilities between "levels of service" (Gopal and Majidzadeh

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TABLE 1. Definitions of Condition Ratings

Rating (1)	Meaning (2)
7	New condition
6	Used to shade between 7 and 5
5	Minor deterioration
4	Used to shade between 5 and 3
3	Serious deterioration
2	Used to shade between 3 and 1
1	Potentially hazardous

1991), and a study of service life prediction (Jiang and Sinha 1989). A study at Princeton University (McCalmont 1990) has determined the overall bridge deterioration for a data base of approximately 2,000 bridges, differentiating between steel and concrete construction. In that study and others (Jiang and Sinha 1989), bridge condition ratings were considered Markovian states.

The present study continues the use of one-to-one correspondence between rating and state. In this case, the data base contains 850 bridges with a total of 2,000 individual spans. All bridges are located in lower suburban New York State. Forty percent of the bridges are made of concrete, and the rest are steel. The oldest structure was built in 1840 and the most recent one in 1990. The majority of the bridges were constructed between 1930 and 1970. Only the bridges built after 1900 were used in the analyses. When available, the "date of the last major construction" was used to calculate the age of the bridge.

This paper is divided into three sections. The first section describes methods for determining the Markovian transition matrices for types of bridges and types of elements and summarizes the results of applying these methods to the population. The analysis in this section assumes that the deterioration of each element is independent of all other bridge components. Section two of the paper considers the interaction between the rate of aging of correlated components; that is, the dependence of one element's Markovian transition matrix on another element's condition rating. The third section gives examples of how the Markovian approach can be used in the assessment of a single bridge and on an entire population of bridges, by assigning a cost or risk to a bridge for each possible condition. Throughout the paper, the analysis of "general recommendation" is based on the overall bridge population, while studies of elemental "condition ratings" are based on data for spans.

SECTION 1: UNCORRELATED ELEMENTS

A typical Markov Chain for stationary bridge deterioration is shown in (1) (seven discrete states are used so that the analysis is consistent with the existing rating scale).

$$\mathbf{T} = \begin{bmatrix} T_{77} & T_{76} & T_{75} & T_{74} & T_{73} & T_{72} & T_{71} \\ 0 & T_{66} & T_{65} & T_{64} & T_{63} & T_{62} & T_{61} \\ 0 & 0 & T_{55} & T_{54} & T_{53} & T_{52} & T_{51} \\ 0 & 0 & 0 & T_{44} & T_{43} & T_{42} & T_{41} \\ 0 & 0 & 0 & 0 & T_{33} & T_{32} & T_{31} \\ 0 & 0 & 0 & 0 & 0 & T_{22} & T_{21} \\ 0 & 0 & 0 & 0 & 0 & 0 & T_{11} \end{bmatrix} \quad (1)$$

where T_{ij} = the probability of an element decaying from state i to state j in one year. Note that all T_{ij} terms where j is greater than i are zero because the condition cannot improve without intervention. There are two additional observations that help simplify (1). First, the analysis of deterioration data for New Jersey bridges showed that the probability of a bridge element decaying by more than one state in two years is negligible (McCalmont 1990). This stems, in part, from the fact that the condition scale is discretized into a sufficiently small number of states. The present study attempts to determine one-year transition probabilities; therefore, the probabilities for two (or more) state jumps are also negligible. Second, the rows of the Markovian transition matrix must sum to one. It follows that only the seven T_{ii} terms are needed to fully define a particular MTM. Eq. (2) shows the form of the MTM used in this study:

$$\mathbf{T} = \begin{bmatrix} T_{77} & 1 - T_{77} & 0 & 0 & 0 & 0 & 0 \\ 0 & T_{66} & 1 - T_{66} & 0 & 0 & 0 & 0 \\ 0 & 0 & T_{55} & 1 - T_{55} & 0 & 0 & 0 \\ 0 & 0 & 0 & T_{44} & 1 - T_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & T_{33} & 1 - T_{33} & 0 \\ 0 & 0 & 0 & 0 & 0 & T_{22} & 1 - T_{22} \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad \ldots\ldots (2)$$

The last term T_{11} = 1, because the condition rating cannot get any worse (or better) and remains trapped in this state. Assuming a stationary process, and given an initial distribution, $\mathbf{q}_0$, the distribution of the condition rating in year n can be found as:

$$\mathbf{q}_n = \mathbf{q}_0\mathbf{T}^n \quad \ldots\ldots (3)$$

Determining Transition Probabilities

Two methods were used to determine the values of the terms T_{77}–T_{22}. The first approach minimizes the summation of the squared difference between the relative frequency and the discrete distribution found from (3). Each term in the total error is weighted by the number of bridges of age n. This method is written as a nonlinear program:

$$\min \sum_{i=1}^{7} \sum_{n=1}^{N} (\mathbf{f}_{i,n} - \mathbf{q}_0\mathbf{T}^n)_i^2 C(n) \quad \ldots\ldots (4a)$$

subject to

$$0 < T_{kk} < 1 \qquad k = 2, \ldots, 7 \quad \ldots\ldots (4b)$$

where $\mathbf{q}_0$ = initial distribution (1, 0, 0, 0, 0, 0, 0); $\mathbf{f}_{i,n}$ = relative frequency of bridges in state i at age n; $\mathbf{T}$ = the matrix from (2); N = number of years of data available; and $C(n)$ = number of bridges of age n. The program was solved by successive line minimizations in the T_{77}–T_{22} directions, using the bisection method.

The second approach (McCalmont 1990) evolves from the previous one, by taking advantage of the fact that $(\mathbf{q}_0\mathbf{T}^n)_i$ is independent of T_{ji} if j is less than i. In other words, the probability of being in state 6 after some time n, depends only on T_{77} and T_{66}, and not on T_{55}–T_{22}. Therefore, T_{77}–T_{22} can be found successively by minimizing the mean-square error for each row.

$$\min \sum_{n=1}^{N} [\mathbf{f}_{i,n} - (\mathbf{q}_0\mathbf{T}^n)_i]^2 C(n) \qquad \text{for } i = 7, 6, \ldots, 2 \qquad (5)$$

where each minimization is only in one variable.

Jiang and Sinha (1989) also used nonlinear programming to estimate transition probabilities for their service-life-prediction model, but they minimized the absolute difference between the expected value of the condition rating from the Markov chain and the actual average condition rating from the data base.

Results

The values of the Markovian probabilities extracted from the New York data base are summarized in Tables 2–4. Each table has two values of T_{ii} for each element: The upper value was obtained with the overall minimum method, and the lower value with the line-by-line minimization method.

TABLE 2. Markovian Transition Matrices for General Recommendation

Description (1)	Count (2)	T_{77} (3)	T_{66} (4)	T_{55} (5)	T_{44} (6)	T_{33} (7)	T_{22} (8)	Percentage (<3) (9)	Percentage (<5) (10)
Concrete									
Simple	214	0.950	0.977	0.987	0.998	0.999	0.999	0.50	26.14
Continuous	54	0.893	0.957	0.976	0.973	0.999	0.945	3.01	57.52
PS	36	0.827	0.997	0.971	0.895	0.915	0.945	1.04	7.01
Deck arch	48	0.958	0.969	0.987	0.999	0.909	0.947	0.02	29.34
Culvert	36	0.893	0.967	0.968	0.997	0.829	0.904	0.39	48.65
Frame	86	0.000	0.000	0.000	0.970	0.949	0.938	56.06	100.00
Steel	456	0.886	0.961	0.974	0.982	0.989	0.977	2.36	57.78
All	830	0.862	0.966	0.981	0.982	0.989	0.974	1.63	53.37

TABLE 3. Elemental-Condition-Rating Markov Chains

Description (1)	Count (2)	T_{77} (3)	T_{66} (4)	T_{55} (5)	T_{44} (6)	T_{33} (7)	T_{22} (8)	Percentage (<3) (9)	Percentage (<5) (10)
Footing	911	0.962	0.943	0.981	0.972	0.956	0.963	1.52	40.55
		0.958	0.933	0.981	0.962	0.881	0.681	2.38	47.00
Column	802	0.942	0.943	0.979	0.964	0.954	0.921	2.84	52.10
		0.936	0.931	0.978	0.955	0.892	0.000	4.31	60.12
Stem	295	0.954	0.956	0.986	0.978	0.982	0.942	0.89	39.34
		0.951	0.957	0.985	0.971	0.950	0.000	1.24	40.28
Top of cap	807	0.949	0.946	0.982	0.957	0.948	0.913	2.49	47.17
		0.947	0.943	0.981	0.944	0.981	0.000	3.41	49.63
Pedestal	880	0.954	0.950	0.983	0.972	0.938	0.943	1.45	42.49
		0.950	0.948	0.983	0.961	0.807	0.000	2.10	45.66
Erosion	131	0.000	0.000	0.979	0.950	0.944	0.952	22.00	100.00
		0.000	0.007	0.957	0.634	0.000	0.000	66.89	100.00
Secondary Member	1,240	0.970	0.965	0.981	0.983	0.954	0.974	0.55	25.34
		0.968	0.968	0.982	0.985	0.965	1.000	0.46	24.93

TABLE 4. Elemental Condition Rating Markov Chains, with Different Types

Description (1)	Count (2)	T_{77} (3)	T_{66} (4)	T_{55} (5)	T_{44} (6)	T_{33} (7)	T_{22} (8)	Percentage (<3) (9)	Percentage (<5) (10)
Prime member	1,958	0.963	0.967	0.981	0.978	0.983	0.999	0.78	28.21
		0.962	0.967	0.982	0.984	1.000	1.000	0.57	28.72
Concrete	492	0.948	0.966	0.983	0.975	0.985	0.999	1.07	35.72
		0.946	0.966	0.983	0.975	0.979	0.828	1.16	36.50
Steel	1,294	0.965	0.968	0.978	0.979	0.978	1.000	0.80	26.56
		0.965	0.968	0.980	0.986	1.000	1.000	0.50	26.55
Wearing surface	1,784	0.937	0.911	0.973	0.968	0.956	0.956	4.57	66.19
		0.926	0.893	0.968	0.950	0.871	0.061	9.21	75.22
AC	332	0.860	0.909	0.972	0.961	0.957	0.957	8.89	85.68
		0.823	0.900	0.967	0.935	0.880	0.000	16.96	90.63
Monolithic	395	0.000	0.947	0.915	0.953	0.941	0.958	24.16	79.39
		0.963	0.971	0.954	0.930	0.699	0.633	3.72	25.68
Portland concrete	581	0.715	0.923	0.962	0.966	0.962	0.952	11.86	87.62
		0.874	0.891	0.958	0.952	0.905	0.000	15.05	88.04
Structural deck	1,420	0.940	0.945	0.978	0.971	0.974	0.960	2.47	52.03
		0.938	0.939	0.979	0.968	0.954	0.598	2.83	55.82
Cast-in-place epoxy bars	131	0.942	0.976	0.991	0.989	0.888	0.933	0.22	29.45
		0.948	0.975	0.991	0.980	0.000	0.000	0.03	28.54
Cast-in-place uncoated bars	1,141	0.937	0.940	0.971	0.974	0.981	0.961	3.14	55.79
		0.937	0.935	0.975	0.976	0.977	0.815	2.73	58.04
Timber	97	0.977	0.923	0.983	0.957	0.975	0.951	0.23	5.28
		0.000	0.946	0.979	0.922	0.877	0.000	12.00	80.00
Joints	611	0.000	0.964	0.888	0.915	0.974	0.978	30.14	65.46
		0.820	0.842	0.860	0.952	0.952	0.654	40.61	97.16
Expansion	516	0.000	0.902	0.888	0.914	0.974	0.983	54.85	94.97
		0.819	0.835	0.858	0.903	0.952	0.703	40.36	79.36

Armored compression	45	0.892	0.917	0.949	0.969	0.941	0.932	9.83	78.66
seal		0.917	0.909	0.950	0.964	0.860	0.000	9.93	74.75
Compression seal	31	0.870	0.840	0.992	0.731	1.000	0.863	9.88	94.14
		0.870	0.833	0.994	0.898	1.000	0.000	5.36	94.54
Filled elastic	273	0.184	0.867	0.851	0.916	0.979	1.000	66.08	98.34
		0.809	0.789	0.875	0.910	0.967	0.990	61.45	98.95
Fixed	96	0.000	0.000	0.000	0.960	0.980	0.972	66.78	100.00
		0.000	0.914	0.889	0.934	0.953	0.000	44.91	92.63
Armored compressed seal	26	0.000	0.989	0.969	0.937	0.943	0.960	4.08	27.44
		0.000	0.987	0.949	0.000	0.000	0.000	14.86	31.57
Filled elastic	51	0.000	0.907	0.833	0.925	0.993	0.908	57.15	94.10
		0.000	0.904	0.821	0.944	0.997	0.000	50.33	94.64

Rare elements in the data base, or components with vague description such as "other" were omitted. Ill-formed results obtained with sufficient data are included with explanations. The values of T_{22} and T_{33} should be used with caution because there are few elements that are in such conditions.

Results for the overall assessment of the bridge "general recommendation" are shown in Table 2. Transition probabilities were determined for all bridges, all concrete bridges and all steel bridges. Further grouping includes concrete nonprestressed, concrete prestressed, steel painted, and steel unpainted. Table 3 shows transition probabilities for generic bridge elements or characteristics that could not be subgrouped because of insufficient data; e.g., footings, columns, and erosion status. Table 4 shows the results for elements that were well supported in the data base. Four elements were examined: primary structure, wearing surface, structural deck, and deck joints.

A meaningful approach to compare two Markov chains is to define a single probability that represents the overall decay rate. Two such values were selected: One is the probability that the condition rating is less than 3 after 30 years, and the other is the probability that the condition rating is less than 5 after 30 years. The probabilities for these threshold ratings can be interpreted as the percent of elements or bridges needing major rehabilitation or minor repair, respectively. These two values are also obtained in Tables 2, 3, and 4.

The Markov chains can also be compared graphically by plotting the expected value of the condition rating versus age. The expected value can be found as:

$$CR_n^* = \sum_{i=7}^{1} (q_0\mathbf{T}^n)_i i \quad \text{(6)}$$

Fig. 1 shows the decay rate of steel structures as observed from the data base, and as predicted by the Markovian model. This is a plot of expected value of condition rating versus time. Because the variance increases with time, 10% and 90% are also shown. However, the lines for the percentiles rarely fall at a discrete state, the value of condition rating for a given percentile is interpolated from the bracketing probabilities.

Ill-formed Markovian transition matrices have 0 in the higher condition

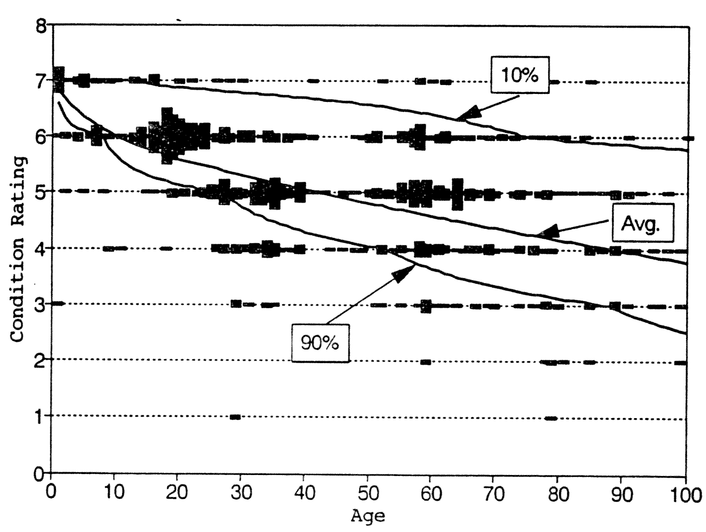

FIG. 1. Deterioration of Steel Structure: Age versus Condition Rating

ratings and 1.0 in the lower. A value of zero for T_{77} indicates that there were very few or no elements with a 7 rating. Therefore, the least mean-square error occurs when the element deteriorates to state 6 as rapidly as possible, i.e., the probability of staying in state 7 is 0.0, thus one year lowers the rating to 6. Similarly, a value of one for T_{33} occurs when there are no elements in state 2, and state 3 becomes an artificially trapping state. In this case, the value T_{22} is meaningless because the mean-square error does not respond to the T_{22} variable. The line-by-line minimization may lead to ill-formed values of 1 or 0 in the lower states. Further work is needed to assess the advantages and limitations of the line-by-line minimization as compared to the overall optimization.

Section 2: Correlated Elements

If elements A and B interact in a structure such that the condition of B effects the decay rate of A, then A must have a nonstationary Markovian transition matrix, in which the terms $\mathbf{T}_{ii}$ decrease with time. (It is assumed the lower rating of B implies a faster decay of A). For example, due to water infiltration the decay rate of the primary structure will depend on the condition of the deck joints. The resulting decay of the primary structure will be a nonstationary Markov chain, because the transition probabilities of the primary structure are decreasing as the joint decays.

Let's define $\mathbf{T}a^j_{ii}$ as the probability of A staying in state i if B is in state j. Then, given a distribution of B, qb^n_j, the transition probability, Ta^*_{ii} in year n is found as:

$$ {}_n\mathbf{T}a^*_{ii} = \sum_{j=1}^{7} \mathbf{T}a^j_{ii} qb^n_j \qquad \text{for } i = 1, \ldots, 7 \qquad (7)$$

The distribution for A at time n is now found from the deterioration matrices for each year, $\mathbf{T}^*_n$.

$$ \mathbf{q}a_n = \mathbf{q}a_0 \mathbf{T}^*_1 \mathbf{T}^*_2 \ldots \mathbf{T}^*_n \qquad (8)$$

The parameters for this model can be found by minimizing the mean-square error, as in (4), however a new $\mathbf{T}^*$ must be found each year from the previously known $\mathbf{T}_B$. Because each state of B has $\mathbf{T}a^j_{ii}$ for each state in A, there are 49 (7 × 7) probabilities that are free in the optimization. The problem can be simplified by reducing the condition rating of B to two states; good, i.e., functioning as designed, and bad, i.e., not functioning as designed. Then, two probabilities can be found at any time n:

$$ P_{\text{good}} = qb_7 + qb_6 + qb_5 \qquad (9)$$

$$ P_{\text{bad}} = qb_4 + qb_3 + qb_2 + qb_1 = 1 - P_{\text{good}} \qquad (10)$$

Now T^*_{ii} can be expressed as

$$ {}_nT^*_{ii} = T_{ii,B=\text{good}} P^n_{\text{good}} + T_{ii,B=\text{bad}} P^n_{\text{bad}} \qquad (11)$$

and only 14 terms need to be found in the minimization. The nonstationarity of T^*_{ii} arises from the fact that P_{good} is decreasing and P_{bad} is increasing with time, and $T_{ii,B=\text{good}}$ is greater than $T_{ii,B=\text{bad}}$.

Fictitious data for a pair of bridge elements were simulated from known transition matrices, to test the method for finding nonstationarity. Both elements were initialized to be in state 7 and were deteriorated with the

Markovian transition matrices shown herein. It is assumed that the condition of element B influences the decay rate of element A as follows:

$$\mathbf{T}_B = \begin{bmatrix} 0.9 & 0.1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.9 & 0.1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.9 & 0.1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.9 & 0.1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.9 & 0.1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.9 & 0.1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1.0 \end{bmatrix}$$

The transition matrix for A given that the condition rating of B > 5 (good)

$$\mathbf{T}_A = \begin{bmatrix} 0.9 & 0.1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.9 & 0.1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.9 & 0.1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.9 & 0.1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.9 & 0.1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.9 & 0.1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1.0 \end{bmatrix}$$

The transition matrix for A given that the condition rating of B < 5 (bad)

$$\mathbf{T}_A = \begin{bmatrix} 0.8 & 0.2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.8 & 0.2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.8 & 0.2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.8 & 0.2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.8 & 0.2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.8 & 0.2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1.0 \end{bmatrix}$$

The results of applying the minimization method on the simulated data are summarized in Table 5, together with the results for the noncorrelated method, so that minima can be compared. The analysis assuming correlation shows a lower minimum, indicating a better fit than the uncorrelated method.

The New York data base was evaluated to identify correlations between the decay rates of different components. Table 6 shows the results of the correlated analysis of the primary structure when it is assumed to be dependent on the condition of the deck joints. Predicted deterioration rates

TABLE 5. Results of Nonstationary Analysis of Simulated Data

	Uncorrelated Analysis		Correlated Analysis of A	
T (1)	A (2)	B (3)	A given B = good (4)	A given B = bad (5)
7	0.881	0.886	0.882	0.869
6	0.879	0.895	0.885	0.798
5	0.865	0.894	0.879	0.778
4	0.863	0.892	0.880	0.797
3	0.862	0.899	0.875	0.808
2	0.869	0.889	0.884	0.811
Minimum	42.72	46.62	36.28	36.28

TABLE 6. Results of Nonstationary Analysis of Real Data

Transition values (1)	T_{77} (2)	T_{66} (3)	T_{55} (4)	T_{44} (5)	T_{33} (6)	T_{22} (7)	min (8)
Primary steel if joint CR > 4	0.991	0.983	0.999	0.999	0.999	—	77.83
Primary steel if joint CR ≤ 4	0.717	0.889	0.896	0.780	—	—	
Primary concrete if joint CR > 4	0.968	0.967	0.999	0.999	0.999	—	59.87
Primary concrete if joint CR ≤ 4	0.728	0.903	0.873	0.889	0.617	—	

Note: CR = condition rating.

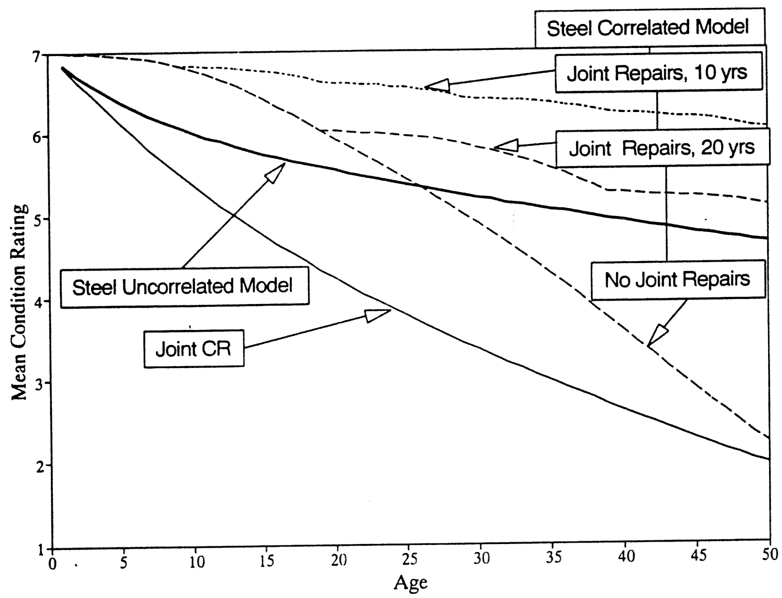

FIG. 2. Deterioration of Joints and Steel Structure: Correlated and Noncorrelated Analyses

of the joints and the steel structures are shown in Fig. 2, based on results of the correlated and uncorrelated analyses. The decay rates of the steel structure when joints are repaired every 10 and 20 years are also shown. The decay rate predicted from the uncorrelated analysis involves standard maintenance of the joints and structure. The corrected prediction of the correlated analysis, taking into consideration joint repair, leads to results similar to the predictions of the uncorrelated analysis.

SECTION 3: APPLICATIONS OF MARKOVIAN APPROACH

Population of Bridges

Markov chains can be used to predict the future distribution of condition ratings of a group of bridges. Table 7, column 1 shows the current number of steel bridges in each condition, in the New York data base. The remaining columns show the predicted distributions for the next five years, found by successively applying the Markov chain obtained from the data base. Be-

TABLE 7. Predicted Distributions of Steel Bridges (Values Shown are Actual Number of Bridges)

Condition rating (1)	Year					
	0 (now) (2)	1 (3)	2 (4)	3 (5)	4 (6)	5 (7)
7	38	32	27	23	19	16
6	176	174	172	170	167	163
5	155	157	160	163	165	167
4	94	95	97	99	102	104
3	40	41	43	44	46	47
2	7	7	7	8	8	9
1	3	3	3	3	3	5
Average CR	5.087	5.069	5.056	5.032	5.014	4.998

Note: CR = condition rating.

cause the number of bridges is constant, the Markov chains can be applied to the relative frequency.

Markov chains can also be used to model the effect of bridge repairs. For example, the repair policy "replace 10% of the bridges in condition 3 or less each year" can be modeled with the following Markovian transition matrix

$$\mathbf{R} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0.1 & 0 & 0 & 0 & 0.9 & 0 & 0 \\ 0.1 & 0 & 0 & 0 & 0 & 0.9 & 0 \\ 0.1 & 0 & 0 & 0 & 0 & 0 & 0.9 \end{bmatrix} \quad \text{(12)}$$

This is an identify matrix except for the last three rows, which have the percentage of bridges to repair in the first column of the rows corresponding to conditions 3, 2, and 1; the complement appears on the main diagonal. The underlying assumption is that the condition rating of the repaired bridges jumps to state 7, given it is presently in condition 3, 2, or 1.

The combined effect of natural deterioration and repair policy can be expressed as **TR**. Then, the distribution in any year expressed as

$$q_n = q_0(\mathbf{TR})^n \quad \text{(13)}$$

The upper part of Table 8 shows the predicted future distributions of bridges when the 10% repair policy is implemented. The lower part shows the results of implementing a 20% repair policy of bridges in condition 3 or lower. The average condition rating for the whole population is shown and can be compared with the evolution of the average condition rating for deterioration without repairs (Table 7). It can be concluded that slightly less than 10% of the bridges in condition 3 or lower should be fixed each year to maintain the same average condition rating in the future, for this network.

Uses of Markovian Approach on a Single Bridge

Markovian analysis can also be used to predict future cost or risks for a single bridge. If each condition rating i has some maintenance cost C_i associated with it, then the expected value of this cost can be found.

$$\text{cost} = \sum_{i=1}^{7} q_i C_i \quad \text{(14)}$$

When separate costs or risks are known for each of the N_e elements in the bridge, the total expected value of the cost can be found as

$$\text{total cost} = \sum_{i=1}^{7} \sum_{j=1}^{N_e} q_{ij} C_{ij} \quad \text{(15)}$$

Risk can be substituted directly into (14), where risk can be the probability of events such as failure or unserviceability. However, when using risk in (15), the summation over all elements is no longer valid because the elemental total probabilities must be combined by systems reliability methods, taking into consideration the structural arrangement, i.e., series, parallel, or any combination.

In either case, cost or risk, repairs can be modeled by forcing the distribution back to state 7, regardless of the current state (this assumption was made in the previous section). This operation can be done systematically with the following Markov transition matrix

$$\mathbf{R} = \begin{bmatrix} 1.0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1.0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1.0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1.0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1.0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1.0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1.0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{(16)}$$

Uncertainties of all forms, including subjective assessment of the quality of the repair, can be captured in **R**. For example, a 90% confidence that a repair will return an element to state 7 can be written as

$$\mathbf{R} = \begin{bmatrix} 0.9 & 0.1 & 0 & 0 & 0 & 0 & 0 \\ 0.9 & 0.1 & 0 & 0 & 0 & 0 & 0 \\ 0.9 & 0.1 & 0 & 0 & 0 & 0 & 0 \\ 0.9 & 0.1 & 0 & 0 & 0 & 0 & 0 \\ 0.9 & 0.1 & 0 & 0 & 0 & 0 & 0 \\ 0.9 & 0.1 & 0 & 0 & 0 & 0 & 0 \\ 0.9 & 0.1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{(17)}$$

Then, the expected cost C_j for element j can now be found by

$$C_j = \sum_{i=1}^{7} c_{i,j} (q_0 \mathbf{t}^n \mathbf{R} \mathbf{T}^m)_i \quad \text{(18)}$$

where q_0 = initial probability distribution of the condition rating; $\mathbf{T}$ = the appropriate transition matrix; $\mathbf{R}$ = the repair transition matrix; $c_{i,j}$ = the cost of element j in condition i; n = number of years before the repair; and m = number of years after the repair.

The elemental cost is then summed over all elements to obtain the total repair cost. This can be repeated with elements being repaired separately or in combinations, for any given year and for each bridge. The combination

TABLE 8. Predicted Distributions of Steel Bridges with 10% and 20% Replacement Policy (Values Shown are Actual Number of Bridges)

Condition rating (1)	Year 0 (now) (2)	1 (3)	2 (4)	3 (5)	4 (6)	5 (7)
			(a) 10% Plan			
7	38	37	36	35	34	34
6	176	179	173	172	171	169
5	155	157	160	163	165	168
4	94	95	97	99	107	104
3	40	37	35	33	31	29
2	7	6	6	6	5	5
1	3	3	3	2	2	2
Average CR	5.087	5.090	5.092	5.093	5.094	5.094
			(b) 20% Plan			
7	38	42	44	46	45	43
6	176	174	174	174	175	175
5	155	157	160	163	165	168
4	94	95	97	109	102	104
3	40	33	27	24	20	17
2	7	6	5	4	4	3
1	3	2	2	2	1	1
Average CR	5.087	5.112	5.131	5.146	5.158	5.166

Note: CR = condition rating.

of repairs for all the networks, which has a minimum total cost (or risk), can be found by integer programming or other optimizing methods.

Conclusions

Two methods for the determination of transition probabilities were described and tested. Neither method prevails by the quality of results, however, the line-by-line minimization involves much less computational effort. Both methods are equally affected by the lack of data. Because the "date of the last major construction" was used to determine the age of the structures, repairs that have not updated "age" show the weakest result, i.e., deck joints.

The results for the correlated elements would be improved if two consecutive years of data were available. With this data, the relative frequencies could be easily found, without the need for minimization, and the true condition of the joints would be known. However, a distinct increase in the decay rate can still be seen by the minimization method.

Markov transition matrices can be used to model the aging of a bridge network, and the aging of independent and dependent bridge components. Repair policies and even subjective estimates can be implemented, creating the basis for probability-based bridge management approaches.

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Appendix. References

"*Bridge inspection manual*." (1982). New York State Dept. of Trans., Structures Design and Construction Div., New York, N.Y.

"*Bridge inventory manual*." (1982). New York State Dept. of Trans., Structures Design and Construction Div., New York, N.Y.

Gopal, S., and Majidzadeh, K. (1991). "Application of Markov decision process to level-of-service based maintenance systems." *TRB Annual Meeting*, Transp. Res. Board, Nat. Res. Council, Washington, D.C.

Jiang, Y., and Sinha, K. (1989). "Bridge service life prediction model using Markov chain." *Transportation Research Record 1223*, Transp. Res. Board, Nat. Res. Council, Washington, D.C., 24–30.

Karin, S., and Taylor, H. (1975). *A first course in stochastic processes*. 2nd Ed., Academic Press, New York, N.Y.

McCalmont, D. (1990). "A Markovian model of bridge deterioration," Bachelor of Science in Engineering thesis, Princeton University, Princeton, N.J.