Soil Compressibility Models for a Wide Stress Range

Song-Hun Chong[1] and J. Carlos Santamarina[2]

Abstract: Soil compressibility models with physically correct asymptotic void ratios are required to analyze situations that involve a wide stress range. Previously suggested models and other functions are adapted to satisfy asymptotic void ratios at low and high stress levels; all updated models involve four parameters. Compiled consolidation data for remolded and natural clays are used to test the models and to develop correlations between model parameters and index properties. Models can adequately fit soil compression data for a wide range of stresses and soil types; in particular, models that involve the power of the stress σ'^{β} display higher flexibility to capture the brittle response of some natural soils. The use of a single continuous function avoids numerical discontinuities or the need for ad hoc procedures to determine the yield stress. The tangent stiffness—readily computed for all models—should not be mistaken for the *small-strain* constant-fabric stiffness. **DOI: 10.1061/(ASCE)GT.1943-5606.0001482.**

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Introduction

Soils subjected to either isotropic compression or k_o (where k_o is under zero lateral strain condition) compression experience volume contraction. Contraction depends on soil type, formation history, diagenesis, prior stress history, porosity, and stress conditions.

A soil compressibility model used for settlement analysis often needs to justify the data in a relatively narrow stress range. However, many geotechnical problems involve soils subjected to either extremely low or extremely high effective stress or a wide effective stress range. Examples include self-weight consolidation (Been and Sills 1981; Cargill 1984; Bartholomeeusen et al. 2002; Stark et al. 2005), seafloor engineering [suction casings (Houlsby et al. 2005) and skirted foundation (Bransby and Randolph 1998)], gradual movement of pipelines resting on the seafloor and lakebeds (Krost et al. 2011; Randolph et al. 2011), pile tips (Yang et al. 2010; Tsuha et al. 2012), blast loads (Wang et al. 2005), methane recovery by depressurization from hydrate bearing sediments, and filter-cake formation in drilling mud (Sherwood and Meeten 1997).

Soil compressibility models are sought in this study to analyze field conditions over a wide stress range. Models must be able to fit compressibility data for diverse soils, have physically correct asymptotic values at low stress $\sigma' \rightarrow 0$ and high stresses $\sigma' \rightarrow \infty$, and involve a small number of physically meaningful parameters (Ockham's criterion).

Compressibility: Stress Regimes

Soil compressibility is briefly reviewed next. Three stress regimes are tentatively identified in reference to the standard stress range in common geotechnical applications, namely 10 kPa $< \sigma'_z <$ 1 MPa.

Low Stress Regime ($\sigma'_z <$ 10 kPa)

Individual grains or flocs form a granular skeleton with a characteristic finite porosity that depends on grain geometry and pore fluid chemistry (Klein and Santamarina 2005; Palomino and Santamarina 2005). Compressibility at low stress reflects the formation fabric and postdepositional diagenetic changes triggered by preloading, moisture fluctuations, thermal history, fluid-mineral interaction, dissolution, and reprecipitation (Mitchell 1956; Burland 1990; Santamarina et al. 2001; Rinaldi and Santamarina 2008).

Intermediate Stress Regime (10 kPa $< \sigma'_z <$ 1 MPa)

Soil compression in this stress regime remains affected by formation conditions such as initial water content (Hong et al. 2010, 2012) and temperature (Campanella and Mitchell 1968; Baldi et al. 1988; Leroueil 1996; Sultan et al. 2002), and is affected by diagenetic processes such as cementation and aging (Mesri et al. 1975; Schmertmann 1983, 1984, 1991). This stress regime is of main interest to classical geotechnical practice; therefore, many studies have explored correlations between compressibility and index properties. In particular, the compression index Cc has a strong correlation with the liquid limit LL, or the void ratio at the liquid limit e_{LL} (Skempton 1944; Terzaghi and Peck 1948; Burland 1990; Sridharan and Nagaraj 2000). Disturbance and/or remolding destructures natural soils, and remolded soils exhibit a compression curve that plots at a lower void ratio than the undisturbed natural soil and with a less pronounced yield stress. Measured consolidation curves are affected by experimental procedures such as sampling disturbance (Casagrande 1936; Terzaghi and Peck 1948; Schmertmann 1955; Rochelle et al. 1981; Hight et al. 1992; Santagata and Germaine 2002), seating and boundary effects, and strain rate (Hanzawa 1989; Leroueil 1996; Leoni et al. 2008).

High Stress Regime ($\sigma'_z >$ 1 MPa)

The void ratio decreases at a gradually lower rate at high stress (Athy 1930; Aplin et al. 1995), and the prevailing deformation mechanisms become particle compliance, pressure dissolution,

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crushing, and creep (Barden 1965; Mesri and Godlewski 1977; Mesri and Castro 1987). Past history loses relevance regardless of the natural or remolded origin of specimens (Terzaghi and Peck 1948; Chilingar and Knight 1960; Skempton 1969; Burland 1990; Hong et al. 2012).

Soil Compressibility Models

Classical e-$\log(\sigma')$ compressibility models and new functions are reviewed in this section. In all cases, the models are generalized to satisfy asymptotic void ratios e_L as $\sigma' \to 0$, and e_H as $\sigma' \to \infty$.

Semi-Logarithmic e-$\log\sigma'$ Models

The classical linear equation in $\log(\sigma')$ is the most common model used in geotechnical engineering (Terzaghi and Peck 1948; Schofield and Wroth 1968)

$$e = e_{\text{ref}} - C_c \log\left(\frac{\sigma'}{\sigma'_{\text{ref}}}\right) \tag{1}$$

where the void ratio e_{ref} corresponds to effective stress $\sigma' = \sigma'_{\text{ref}}$. The normalization stress σ'_{ref} is selected a priori for normalization, such as $\sigma'_{\text{ref}} = 1$ kPa, and it is not a model parameter. Therefore, this function has two model parameters: e_{ref} and C_c; it fits normally consolidated soil data at intermediate stress levels, but predicts $e \to \infty$ as $\sigma'_z \to 0$ and $e < 0$ as $\sigma'_z \to \infty$.

The lower soil compressibility at high stress requires higher order terms, such as the cubic polynomial suggested by Burland (1990) for remolded soil data

$$e = e_{\text{ref}} - \alpha \cdot \log\left(\frac{\sigma'}{\sigma'_{\text{ref}}}\right) + \beta \cdot \left[\log\left(\frac{\sigma'}{\sigma'_{\text{ref}}}\right)\right]^3 \quad (\text{stress range } 10\text{ kPa} < \sigma' < 10\text{ MPa}) \tag{2}$$

in terms of three model parameters e_{ref}, α, and β. In addition, the asymptotic void ratio e_L at low stress can be imposed as a plateau.

Alternatively, the classical semi-logarithmic Terzaghi model can be modified to satisfy asymptotic conditions: $e \to e_L$ at low stress $\sigma' \to 0$, and $e \to e_H$ at high stress $\sigma' \to \infty$

$$e = e_c - C_c \log\left(\frac{1\text{ kPa}}{\sigma' + \sigma'_L} + \frac{1\text{ kPa}}{\sigma'_H}\right)^{-1} \tag{3}$$

where parameters e_c and C_c determine the central trend, and void ratio asymptotes e_L and e_H define stresses σ'_L and σ'_H

$$\sigma'_H = 10^{(e_c - e_H)/C_c} \cdot \text{kPa} \quad \text{when } \sigma' \to \infty \tag{4}$$

$$\sigma'_L = \frac{\sigma'_H}{10^{(e_L - e_H)/C_c} - 1} \quad \text{when } \sigma' \to 0 \tag{5}$$

The generalized Terzaghi model in Eq. (3) involves four model parameters of clear physical meaning.

Models in Terms of $e - \sigma'^{\beta}$

Power function: From gases to soils. Loosely packed small grains bear resemblance to the notion of a gas. Boyle-Mariotte's law ignores the size of molecules, and concludes that pressure and volume are inversely related

$$\text{PV} = \text{constant} \to V = a/P \tag{6}$$

where α = constant. van der Waals corrected this expression to take into consideration the size of molecules and rewrote Boyle's equation in terms of the *contractible* volume V', i.e., the total volume V_t minus the volume *excluded* V_{ex} by the molecules, $V' = V_t - V_{\text{ex}}$. He also considered intermolecular interactions and the additional stress due to uncompensated attraction at the boundaries.

Following a parallel analysis, and taking into consideration electrical attraction and repulsion in fine-grained soils (σ_A–σ_R), a plausible equation for soil compressibility becomes:

$$(\sigma' + \sigma_A - \sigma_R)(V_t - V_s) = \alpha \tag{7}$$

If the volume of solids as V_s is assumed constant, this equation can be written in terms of void ratio $e = (V_t - V_s)/V_s$

$$e = \frac{\alpha'}{\sigma' + \sigma_A - \sigma_R} \tag{8}$$

This inverse relationship between void ratio e and effective stress σ' can be generalized as a four-parameter inverse power function that accommodates the two void ratio asymptotes e_L and e_H

$$e = e_H + (e_L - e_H)\left(\frac{\sigma' + \sigma'_c}{\sigma'_c}\right)^{-\beta} \tag{9}$$

When the applied effective stress equals the characteristic effective stress $\sigma' = \sigma_c$ and $\beta = 1$, the predicted void ratio is the average of the asymptotes $e = (e_L + e_H)/2$. Higher β-exponents cause higher early compressibility at lower stresses. Power-type equations have been suggested in the past (Hansen 1969; Butterfield 1979; Juárez-Badillo 1981; Houlsby and Wroth 1991; Pestana and Whittle 1995).

Exponential: The main characteristic of exponential functions $y = \exp(x)$ is that the rate of change dy/dx is defined by the current state. Sigmoidal and Gompertz functions are special examples (Gompertz 1825; Gregory et al. 2006). The four-parameter Gompertz function can be expressed in terms of stress and void ratio, and adapted to satisfy e_L and e_H as follows:

$$e = e_H + (e_L - e_H) \cdot \exp^{-\left(\frac{\sigma'}{\sigma'_c}\right)^{\beta}} \tag{10}$$

When the exponent $\beta = 1$, the simpler three-paremeter exponential expression is obtained (Cargill 1984); it predicts that the soil will experience 63% of the volume change $e_L - e_H$ when the applied effective stress equals the characteristic effective stress $\sigma' = \sigma'_c$.

Hyperbolic: The hyperbolic model is extensively used in geomechanics to capture the prepeak deviatoric stress versus strain data (Kondner 1963; Duncan and Chang 1970). This model has two parameters: one defines the initial rate of change $dy/dx|_o$, and the other provides the asymptotic value of y as $x \to \infty$. The model can be adapted to capture compressibility data in terms of $\sigma' - e$. The generalized four-parameter hyperbolic model is

$$e = e_L - (e_L - e_H)\frac{1}{1 + \left(\frac{\sigma'_c}{\sigma'}\right)^{\beta}} \tag{11}$$

The simpler hyperbolic model ($\beta = 1$) predicts that the void ratio will reach the intermediate void ratio $e = (e_L + e_H)/2$ when the applied effective stress equals the characteristic stress $\sigma' = \sigma'_c$. Structuration and yield stress can be captured with higher values of the β-exponent.

Arctangent: Other functions that provide S-shaped trends can be adapted to satisfy asymptotic conditions relevant to soil compression data. For example, the arctangent function can be generalized to include the power of the stress σ'^{β} in order to fit more brittle soil responses

$$e = e_{\mathrm{L}} + \frac{2}{\pi}(e_{\mathrm{L}} - e_{\mathrm{H}})\arctan\left[-\left(\frac{\sigma'}{\sigma'_c}\right)^{\beta}\right] \tag{12}$$

in terms of four model parameters e_{L}, e_{H}, the characteristic stress σ'_c, and the β-exponent. For $\beta = 1$, the void ratio reaches $e = (e_{\mathrm{L}} + e_{\mathrm{H}})/2$ when the applied effective stress equals the characteristic stress $\sigma' = \sigma'_c$.

Discussion

Examples

Compression data gathered for a wide stress range are fitted using these models in Fig. 1 for both remolded soils and for natural soils with distinct yield stress; fitting parameters are summarized in Table 1. Data points and fitted models are plotted in both log-linear and linear-linear plots; the last column in Fig. 1 shows computed trends for the tangent constrained modulus M_{tan}. The following can be observed:

- Remolded soils: All four-parameter models addressed here can adequately fit experimental data gathered for remolded soils [Fig. 1(a)]. The tangent stiffness computed with these models shows monotonic stiffening.
- Natural soils: Hyperbolic, arctangent, power, and exponential (albeit to a lesser extent) models written in terms of σ'^{β} approxwimate the brittle response of natural structured soils better than $\log(\sigma')$ models [Fig. 1(b)]. The tangent stiffness computed with hyperbolic and arctangent models shows early softening until the yield stress, followed by compaction-driven stiffening.

Furthermore, the S-shaped models presented earlier are well suited to fit the response of overconsolidated soils as well. These observations are confirmed with multiple cases compiled from the literature.

Correlations—Low-Stress Void Ratio and Compressibility

A database of consolidation tests was compiled from the literature for the purposes of this study [The complete database can be found in Chong (2014).] The classical Terzaghi model was fitted to data gathered with remolded, normally consolidated soils within the available effective stress range. Results show that the void ratio $e_{1\ \mathrm{kPa}}$ is closely related to the void ratio at the liquid limit $e_{\mathrm{LL}} = G_s\mathrm{LL}/100$ (assuming 100% saturation)

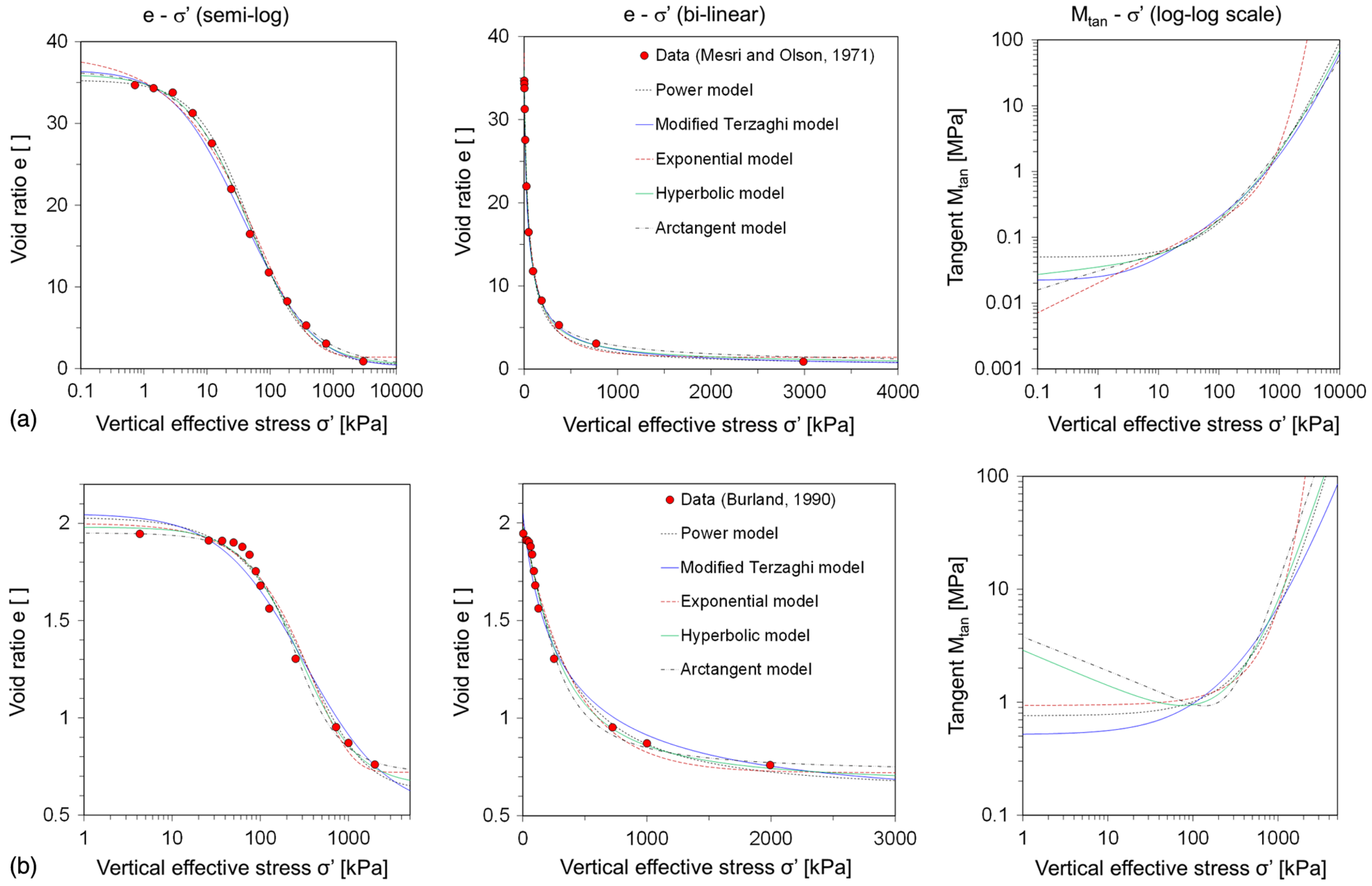

Fig. 1. Wide stress range 1D compression data (k_o, zero lateral strain boundary condition) fitted with different soil compressibility models: (a) remolded clay—sodium montmorillonite at 0.001 N and pH = 7 (data from Mesri and Olson 1971); (b) natural clay—Bothkennar soil from 6.5 m (data from Burland 1990)

$$e_{1\,\mathrm{kPa}} = \frac{5}{4}e_{\mathrm{LL}} = \frac{1}{80}\mathrm{Gs}\cdot\mathrm{LL}$$

$$\text{remolded soils } (28 \text{ cases, SD} = 0.36,\ R^2 = 0.92) \qquad (13)$$

This equation compares well with the e_{LL} versus e_{100} correlation in Burland (1990), as $e_{1\,\mathrm{kPa}} = e_{100\,\mathrm{kPa}} + 2C_c$. Furthermore, a direct link is obtained between the void ratio at 10 kPa and the void ratio at liquid limit

$$e_{10\,\mathrm{kPa}} = e_{\mathrm{LL}} \text{ remolded soils } (28 \text{ cases, SD} = 0.26,\ R^2 = 0.90) \qquad (14)$$

The correlation between compressibility and liquid limit was recognized early on in the field (Skempton 1944; Terzaghi and Peck 1948; Burland 1990). The presented database shows a very similar trend

$$C_c = 0.008\cdot(\mathrm{LL}-5)$$

$$\text{remolded soils } (28 \text{ cases, SD} = 0.11,\ R^2 = 0.90) \qquad (15)$$

Given the $e_{\mathrm{LL}} - e_{1\,\mathrm{kPa}}$ and $C_c - \mathrm{LL}$ correlations, this study explored the relationship between model parameters $e_{1\,\mathrm{kPa}}$ and C_c [Fig. 2(a)]

$$e_{1\,\mathrm{kPa}} = 3.4\cdot C_c + 0.48$$

$$\text{remolded soils } (54 \text{ cases, SD} = 0.19,\ R^2 = 0.96) \qquad (16)$$

A limited set of nine cases was identified to compare natural soils with the same soil after remolding (from Mitchell 1956; Mesri et al. 1975; Burland 1990; Hong et al. 2006; Wesley 2009). Structured natural soils pack at higher void ratio $e_{1\,\mathrm{kPa}}^{\mathrm{natural}} = 1.6\cdot e_{1\,\mathrm{kPa}}^{\mathrm{remolded}}$ and compress more $C_c^{\mathrm{natural}} = 1.7\cdot C_c^{\mathrm{remolded}}$ than the remolded counterparts.

The link between void ratio at low stress and compressibility in $\log(\sigma')$ models emerges in other models too. For example, a measure of compressibility in the hyperbolic model $\sigma_c'/(e_{\mathrm{L}} - e_{\mathrm{H}})$ is linked to the asymptotic low-stress void ratio e_{L} as [Fig. 2(b)]

$$e_{\mathrm{L}} = 15.8\left(\frac{\frac{\sigma_c'}{1\,\mathrm{kPa}}}{e_{\mathrm{L}} - e_{\mathrm{H}}}\right)^{-0.4}$$

$$\text{natural clays } (23 \text{ cases, SD} = 0.4,\ R^2 = 0.84) \qquad (17)$$

Parameter Invertibility

Parameters were selected by least-square fitting the models to the data, $\min[\Sigma(e_i^m - e_i^p)^2]$. The error surface about the optimal parameter set is explored by varying one parameter at a time to assess the invertibility of each parameter. As anticipated, limited data at very low or very high stress results in poor convergence for e_{L} or e_{H}, respectively.

Model parameters can be constrained with a priori data, for example, mineralogy to bound e_{L}, and geological data to restrict e_{H}. Furthermore, the β-exponent is bounded (Table 1), and the low-stress void ratio e_{L} and compressibility are correlated (Fig. 2). These constraints and correlations help to identify a self-consistent set of fitting parameters; furthermore, they suggest an effective model complexity lower than the four unknowns involved in these models.

Void ratio in sedimentary basins: Compressibility models [Eqs. (1)–(3), (9)–(12)] can be integrated to compute void ratio trends versus depth. The nonlinear decrease in void ratio with depth observed in sedimentary basins (e.g., data reported in Aplin et al. 1995) can be properly matched with all models reviewed here.

Table 1. Fitting Parameters (Refer to Data Presented in Fig. 1)

Model	Parameters	Montmorillonite 0.001 N [Fig. 1(a)]	Bothkennar [Fig. 1(b)]
Modified Terzaghi	e_{L}	36.5	2.05
	e_{H}	0.20	0.52
	e_c	52	5.2
	C_c	21.5	1.6
Power	e_{L}	35.1	2.03
	e_{H}	0.40	0.63
	β	1.0	2.0
	σ_c' (kPa)	48	700
Exponential	e_{L}	38.5	2.00
	e_{H}	1.40	0.72
	β	0.55	1.0
	σ_c' (kPa)	70	400
Hyperbolic	e_{L}	36.0	1.98
	e_{H}	0.40	0.65
	β	0.9	1.32
	σ_c' (kPa)	43	280
Arctangent	e_{L}	36.5	1.95
	e_{H}	0.30	0.72
	β	0.7	1.3
	σ_c' (kPa)	42	250

Tangent stiffness: Small-strain versus large-strain. Numerical solutions that use a tangent formulation involve the tangent constrained modulus M

$$M = \frac{\partial\sigma'}{\partial\varepsilon} = \frac{-\partial\sigma'}{\partial e}(1+e) \qquad (18)$$

The void ratio versus stress models discussed earlier can be applied to isotropic loading conditions by replacing $\sigma' \rightarrow p'$ and $e \rightarrow v = 1 + e$. Then, the tangent bulk stiffness K becomes

$$K = \frac{dp'}{d\varepsilon_v} = \frac{-dp'}{dv}v \qquad (19)$$

Equations obtained for either M or K for all compressibility models [Eqs. (1)–(3) and (9)–(12)] can be found in the Supplemental Data.

The tangent constrained modulus M or bulk modulus K computed using Eqs. (18) and (19) (Supplemental Data) are fundamentally different from the small strain stiffness measured at the same $e - \sigma'$ state. The *tangent stiffness* is a mathematical concept that reveals the instantaneous rate of fabric change during a large strain test. By contrast, a *small-strain* perturbation (e.g., shear wave propagation) is a constant fabric measurement of stiffness and is determined by contact deformation. Therefore, the magnitude of tangent stiffness and the trends reported in Fig. 1 should not be associated to small-stress stiffness values.

Settlement computation: Estimation of yield stress: Standard settlement analyses assume recompression ($e_{1\,\mathrm{kPa}}^r$, C_r) and normal compression ($e_{1\,\mathrm{kPa}}^n$, C_c) segments before and after the yield stress σ_y'. Several ad hoc methods have been proposed to determine the yield stress or preconsolidation pressure (Casagrande 1936; Janbu 1969; Pacheco 1970; Sallfors 1975; Butterfield 1979; Becker et al. 1987; Oikawa 1987; Jose et al. 1989; Sridharan et al. 1991; Onitsuka et al. 1995; Grozic et al. 2003; Clementino 2005; Boone 2010; Ku and Mayne 2013). Single function models, such as those compiled and augmented here, can capture the complete compression response and streamline computations.

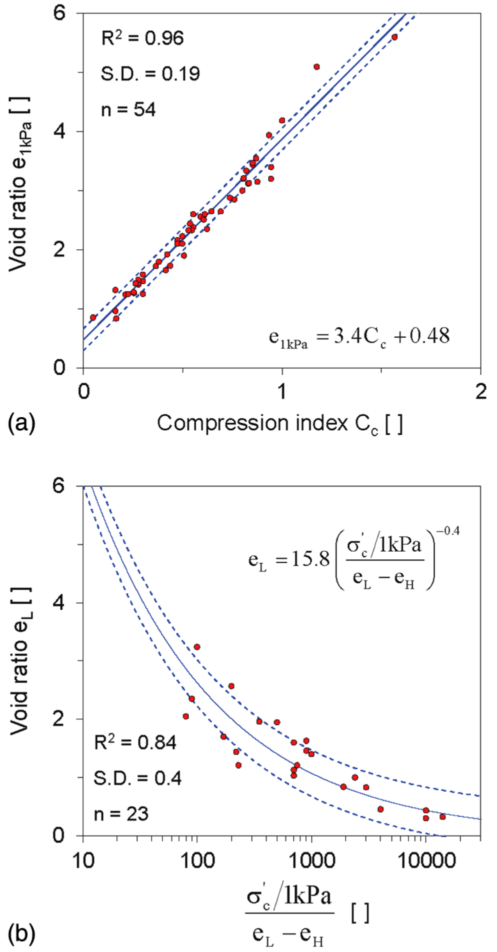

Fig. 2. Correlation between low-stress void ratio and compressibility: (a) classical Terzaghi model as the correlation between void ratio $e_{1\ kPa}$ and C_c for remolded and natural sedimentary clays; (b) hyperbolic function fitted with $\beta = 1$; in both cases, the solid line shows the central trend defined by the equation shown in each frame; dotted lines show the $+/-1$ standard deviation from the central trend

Conclusions

Many geotechnical problems involve either very low, very high, or a wide range of effective stress. In this study, previously suggested compression models and other functions are modified to satisfy asymptotic conditions at low and high stress levels. These models are used to fit the response of normally consolidated remolded soils, structured natural soils, and overconsolidated soils. The following salient observations can be made:

- At least four parameters are required to fit soil compression data gathered in a wide stress range. Physical insight, databases, and the high correlation between compressibility and the void ratio at low stress help to constrain the parameter space to determine a self-consistent set of fitting parameters;
- Models in terms of σ'^{β} (arctangent, hyperbolic, power, and exponential) show more flexibility to capture the compression response of structured natural soils with pronounced brittle transitions at the yield stress;
- The use of a single continuous function to capture soil compressibility data avoids numerical discontinuities, facilitates computing the tangent constrained modulus for numerical methods, and eludes ad hoc procedures to identify the yield stress; and
- The computed tangent constrained modulus should not be mistaken for the instantaneous small-strain modulus measured during the test. The tangent stiffness is a mathematical concept that reveals the instantaneous rate of fabric change during a large strain test. By contrast, a small-strain perturbation test gives a constant fabric stiffness that is determined by contact deformation.

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Supplemental Data

Tables S1 and S2 are available online in the ASCE Library (www.ascelibrary.org).

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