Data Pre-Processing in Cross-Hole Geotomography

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Abstract

The inversion of cross-hole data leading to tomographic imaging magnifies model and measurement errors. This effect couples with limited illumination angles and mixed-determination to render irrelevant images which often resemble the spatial coverage of the measurements. Yet, proper inversions may still be obtained by adding information to the inverse problem. This paper centers on the pre-processing of cross-hole tomographic data to identify the general characteristics of the host medium, the presence of anomalies, and the overall nature of the inverse problem. Several pre-processing strategies are examined within the context of two well-documented case histories. Results confirm the ability of pre-processing to provide foresight about the medium to be imaged and to help select an adequate inversion strategy.

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Introduction

Tomographic imaging is the inversion of boundary measurements to determine the distribution of a parameter within a medium. In the case of tomographic imaging with seismic or electromagnetic waves, transducers are placed on the boundaries of the unknown region, and measured travel times or amplitudes are used to compute velocity or attenuation tomograms.

Tomographic imaging the subsurface has proven to be significantly more complex than initially expected. The remarkable success of tomographic imaging in medical diagnosis and a large number of publications with simulated data showing excellent reconstructions have led to high expectations for geotomography, and made difficult the identification of the critical issues that are faced.

A linear transformation is the simplest mathematical formulation of tomographic imaging. In the case of travel time tomography under the validity of the ray approximation,

$$\underline{t} = \underline{\underline{L}} \cdot \underline{s} \qquad (1)$$

where

$\underline{t}$ (mx1) array; t_i is the i-th <u>measured</u> travel time

$\underline{s}$ (nx1) array; s_j is j-th <u>unknown</u> pixel slowness (slowness = 1/velocity)

$\underline{\underline{L}}$ (mxn) matrix; $L_{i,j}$ is the <u>inferred</u> length traveled by the i-th ray in the j-th pixel.

The purpose of tomography is to obtain the unmeasured values of slowness $\underline{s}$ from the measured values of travel times $\underline{t}$ (a similar formulation can be used for amplitude measurements). A model must be assumed to capture the physics of wave propagation in order to compute the entries of $\underline{\underline{L}}$. The inverted values $\underline{s}$ are used to color the corresponding pixels, rendering the tomographic image.

The entries of the non-negative matrix $\underline{\underline{L}}$ are inferred by ray tracing; this is the forward problem. Differences between the forward model simulator and the physical reality in the subsurface cause modeling errors. For example, straight rays may be used, yet the medium may have a strong velocity gradient causing ray bending. Measured travel times $\underline{t}$ include systematic and accidental errors; these reflect triggering problems, difficulties in the detection of first arrivals, and misinterpretation of records (multi-propagation modes such as P, S and surface waves, multiple travel paths, indirect paths with higher energy, etc.). Measurement and modeling errors render the resulting system of equations inconsistent. Limitations in illumination angles in geotechnical tomography and the uneven spatial coverage of the region add additional difficulty. For example, the L2 error function (sum of the square errors) is not strictly convex in cross-hole tomography. In this case, the solution may be trapped in local minima (Santamarina and Reed 1994).

The geotomographic inverse problem is ill-conditioned, hence its solution is severely hampered by the magnification of measurement and modeling errors. This situation is improved by adding information to the system, such as an initial guess of the solution $\underline{s}_o$ weighting the measurements, or enforcing some property to the solution in the form of a regularization matrix (Morozov 1993; Santamarina and Fratta 1998).

Early experimental studies with cross-hole data obtained within ideal laboratory conditions highlighted the importance of data pre-processing in gaining insight into the characteristics of the medium and the presence of anomalies. This information can be used to properly guide the inversion process. Data pre-processing is the central theme of this paper. Case history data are used to demonstrate different techniques.

Case Histories

Two well-documented case histories are selected for this study. Both cases involve cross-hole transmission measurements of travel time using mechanical waves.

Case 1: High velocity anomaly in homogeneous-isotropic host medium.

An acoustic tomographer was built to simulate the gathering of cross-hole data in the field. The instrumentation frame (1.5m x 1.5m) is held on a horizontal plane, at midheight in the laboratory. Floor and ceiling reflections arrive significantly after the direct arrival, causing no interference. The single propagation mode in acoustic waves and the lack of interference due to the remote location of boundaries facilitates the interpretation of records and the detection of first arrivals.

One side of the frame supports 16 equally spaced capacitor microphones. The source is activated at 16 equally spaced locations along the opposite side, to generate cross-hole data. The signals detected by the microphones are digitized and stored in a PC-based digital storage oscilloscope that is triggered with the source. Data presented in this study corresponds to a 0.46m diameter circular balloon filled with helium and placed at the center of the instrumented frame (Figure 1a). This is a high velocity inclusion.

Case 2: Low velocity anomaly in heterogeneous-anisotropic host medium

The purpose of this tomographic study was to image a tunnel 81m below the surface (Figure 1b; Rechtien and Ballard 1993; see also Rechtien et al. 1995). The dimensions of the tunnel are approximately 2.7m wide and 2.2m high. A sparker source was used to generate signals with a frequency range between 1.4 kHz and 1.7 kHz. A hydrophone was used as the receiver. Seven cross-hole data sets were collected. Each data set was obtained by simultaneously lowering both the source and the receiver in the two parallel vertical holes, 15.2m apart. In one data set, the source and the receiver were kept at the same elevation (θ=0°). The other data sets were obtained by offsetting the elevations of the source and the receiver to generate rays at different inclinations, θ= +45°, +30°, +15°, -15°, -30° and -45°. Measurements were repeated every 0.2 meter. There are 150 travel times in each set.

Data Pre-Processing

Data pre-processing helps the engineer gain insight into the characteristics of the data and the experiment, in order to select the most convenient model and parameters for data inversion. The following procedures are discussed and demonstrated within the context of the two case histories described above: spatial resolution, errors, information content and spatial coverage, average velocity and residuals, characteristics of the host medium (anisotropy, gradual changes in field parameters), and pre-detection of anomalies. Some of these strategies can also be considered while planning the tomographic study to optimize experimental design.

Spatial Resolution (Fresnel's Ellipse and Penetration Depth)

The trade-off between spatial resolution and penetration depth is inherent to tomographic imaging: short wavelength is needed to detect small anomalies, yet long wavelengths must be used to obtain adequate penetration over practical distances.

The wavelength is the spatial scale of wave phenomena. Hence, the selected pixel size should not be much smaller than the wavelength. The wave length λ is estimated with the frequency of the arriving front. Consider the i-th ray, with length L_i and measured travel time t_i. The estimated λ_{est} is

$$\lambda_{est} \approx V_{ave} \cdot T = \frac{L_i}{t_i} \cdot \frac{1}{f} \qquad (2)$$

When the size of inclusions is within the same order of magnitude as the wavelength, propagation must be considered from the point of view of the wave front and scattered energy. Diffraction degrades the quality of tomograms when the linear ray assumption is made: low velocity inclusions are imaged smaller than real size, and high velocity anomalies are imaged larger. Furthermore, diffraction healing adds difficulty to the detection of low velocity anomalies; this is particularly important when the plane of receivers is about two diameters or more away from the inclusion (Potts and Santamarina 1993).

The position of scatterers that affect the wave front arriving at a receiver is also related to the wave length λ. Consider a source S and a receiver R separated by a distance L^{SR}. The Fresnel ellipse is drawn with foci at S and R and cord length $L^{SR}+\lambda/4$ (in the case of multiple reflectors, the cord length becomes $L^{SR}+\lambda/2$). Any anomaly located within this ellipse will generate a reflection that will arrive to the source in phase with the direct wave traveling the straight path L^{SR}. This situation resembles a "thick ray" and should be considered while selecting the separation between transducers during experimental design: transducers that render extensive superposition of the corresponding Fresnel's ellipses do not contribute independent information.

The skin depth is the inverse of the spatial attenuation α [Np/m],

$$S_d = \frac{1}{\alpha} \qquad (3)$$

For mechanical waves, the attenuation coefficient can be estimated from the damping coefficient D, $\alpha=2\pi D/\lambda$. Then,

$$S_d = \frac{1}{\alpha} = \frac{\lambda}{2\pi} \cdot \frac{1}{D} \qquad \textit{Mechanical Waves} \qquad (4)$$

where λ and f are wavelength and frequency, respectively. In general D<5% and the skin depth S_d is several times the wave length; in this case amplitude decay is governed by geometric spreading. In the case of electromagnetic waves, the attenuation coefficient is the real part of the propagation constant. Then, the skin depth can be expressed as,

$$S_d = \frac{1}{\alpha} = \frac{\lambda_o}{2\pi} \cdot \frac{1}{\mathrm{Re}\left\{\sqrt{j\frac{\sigma}{\varepsilon_o\omega}\mu_r - \varepsilon_r\mu_r}\right\}} \qquad \textit{Electromagnetic Waves} \qquad (5)$$

where the wave length $\lambda_o=V_o/f$ and $V_o=3\cdot10^8$ m/s is the speed of light in free space, ε_o is the permittivity of free space, ε_r and μ_r are the relative permittivity and permeability of the medium, and σ its conductivity.

The wavelength in the balloon study is $\lambda_{est} \approx$ 0.15m, which is smaller than the anomaly and compatible with the separation between transducers. The wavelength for the tunnel study is $\lambda_{est} \approx 3$ m and about the tunnel size. The separation between transducers in this case is much smaller than the wavelength; in fact, half of the measurements would have proven sufficient.

Global Information Content

How many linear independent equations are available in the matrix $\underline{\underline{L}}$? While "rank" is a clear mathematical concept, its interpretation is not trivial. For example, the second row in the 2x2 matrix $[(1,0),(1,10^{-10})]$ is almost the same as the first one, yet the rank is 2.

An alternative approach to identify rank deficiency is to compute the singular values of the matrix. The mxn matrix $\underline{\underline{L}}$ can be written as the product of three matrices: $\underline{\underline{L}}=\underline{\underline{U}}\cdot\underline{\underline{\Omega}}\cdot\underline{\underline{V}}^T$ (Golub and Van Loan 1989). The columns of the mxm matrix $\underline{\underline{U}}$ are eigenvectors of $\underline{\underline{L}}\cdot\underline{\underline{L}}^T$ and span the space of the measurements. The columns of the nxn matrix $\underline{\underline{V}}$ are eigenvectors of $\underline{\underline{L}}^T\underline{\underline{L}}$ and span the space of the unknown pixel values. The mxn matrix $\underline{\underline{\Omega}}$ is formed with the eigenvalues ω of $\underline{\underline{L}}^T\underline{\underline{L}}$ (the non-zero eigenvalues are the same as for $\underline{\underline{L}}\cdot\underline{\underline{L}}^T$): the diagonal elements are $\Omega_{i,i}=\omega_i$ and all other entries are $\Omega_{i,j}=0$. The number of non-zero singular values is the best indicator of rank deficiency in the $\underline{\underline{L}}$ matrix. A related difficulty is to decide how small a singular value should be before it stops being considered. The plot of sorted singular values, or spectrum, can be used to facilitate this decision.

Let's consider the cross-hole tomographic experiment with the balloon. The $\underline{\underline{L}}$ matrix is generated assuming straight rays for different pixel resolutions; for example, if the space is discretized in 8-by-8, there are 64 pixels or unknowns. Figure 2a shows the spectra for selected resolutions (the spectrum is the plot of sorted singular values against the index number). Each spectrum is normalized with respect to its largest singular value to facilitate the comparison.

Figure 2b shows the number “p” of significant singular values normalized with respect to the number of measurements or rows m, for each degree of resolution. The value of p is obtained by counting the singular values in the interval [ω_{max}, ω_{max}/100]. This plot suggests that the deficiency between available information (p non-zero singular values) and requested information (m pixel values) becomes pronounced if resolution exceeds 8x8 pixels. Still, higher resolution can be achieved while controlling ill-posedness if additional information can be included in the form of regularization, initial guess, weights, etc.

Spatial Coverage

Civil engineering applications of tomographic imaging often face restrictions in possible illumination angles. This situation causes the uneven spatial coverage of the region under study, mix-determination, and spatial variation of the variance of the solution. The coverage Ψ_j of the j-th pixel can be estimated as the length traversed by all rays in that pixel, which is the sum of the columns in $\underline{\underline{L}}$ (Santamarina 1994),

$$\Psi_j = \sum_{i=1}^{m} L_{ij} \qquad (6)$$

Figures 3a&b show the spatial coverage of a region with side-to-side and top-to-side illuminations. Noise caused by model and data errors tends to be “dumped” in low information regions. Thus, the analyst is well advised to skeptically consider a tomographic image that resembles the corresponding image of spatial coverage.

Spatial coverage and the singular values of $\underline{\underline{L}}$ can be estimated in advance by assuming straight rays. Hence these techniques are helpful while designing optimal experimental configurations.

Heterogeneity and Anisotropy in the Host Medium

Average ray velocities can be estimated before inversion assuming straight rays:

$$\left(V_{ave}\right)_i = \frac{L_i}{t_i} \qquad (7)$$

Polar and spatial variations in average velocity provide clear indications of heterogeneity and anisotropy in the host medium.

Let's consider the tunnel data. Figure 4-a shows the variation of average velocity vs. the average depth of rays [(depth source + depth receiver)/2]. Results are presented for three different ray inclinations. There is a gradual increase in vertical velocity at a rate of ~12(m/s)/m. On the other hand, Figure 4-b shows the variation of average ray velocity vs. the inclination of rays, for three different depths. The observed anisotropy is about 7% between the horizontal and the $\pm 45°$ rays. Clearly, the medium is anisotropic and vertically heterogeneous (velocity increases with depth).

Analysis of Shadows: Anomalies and Errors

The analysis of projections, or "shadows" facilitates detecting the presence of anomalies, their position and size. The analogy is equivalent to illuminating the space with a flashlight and observing the shadow on a projection screen. In order to avoid the effect of path length, the average ray velocity is computed for each source and receiver pair assuming a straight ray path (Equation 7). Shadows are formed by plotting the average ray velocity against the location of corresponding receivers, as schematically shown in Figure 5.

Shadows can be constructed using fan ray paths (from a single source or from a single receiver) or parallel ray paths. They can be back-projected to constrain the location of the anomaly (a technique based on fuzzy logic is described in Santamarina and Fratta 1998; the Fourier slice theorem, back-projection and back-propagation are discussed in

Kak and Slaney 1988; the cross-hole implementation is presented in Witten and Molyneux 1988). The shadows for the balloon case are shown in Figure 6. The gradual shifting of the high-velocity shadow as the source is moved from location s1 to location s16 is distinctly shown.

An important byproduct of this plot is the detection of errors in the measurements. Fluctuations in average velocity are about ± 0.5m/s. On the other hand, the presence of the anomaly increases the average ray velocity from 343m/s to about 360m/s so that ΔV_{ave}=17m/s. Hence, the presence of the balloon (size and velocity contrast) is clearly detectable above the noise level.

Singular value decomposition shows that small singular values magnify the effect of measurement errors during inversion. Hence, it is advantageous to remove measurement errors before proceeding with data inversion. In this context, the analyst may chose to low-pass filter the shadows to remove the high frequency noise. Corrected travel times are computed as $t_i^{corr}=L_i/(V_{ave})^{filtered}$. This approach is suggested only if there is clear evidence that local fluctuations in average ray velocity shadows are the effect of measurement noise.

Time-length Plots: Systematic Errors and Alternative paths

The presence of systematic errors can be investigated with travel length vs. travel time plots. The plot of travel time vs. ray length should define a straight line through the origin when data was gathered for a homogeneous isotropic medium. Hence, the zero-length time offset is a measure of a systematic error in the data, such as trigger delay. This analysis is weakened when there are multiple anomalies, large anomalies, or if all rays are of about the same length. For example, rays in cross-hole data gathered at angles between +45° and -45° vary in length between the borehole spacing B and 1.4B. Figure 7 shows time-length plots for the balloon data. Most rays are affected by the balloon and a blind regression would be misleading.

There is an important advantage in time-length plots: the identification of different "events" which are evidenced by characteristic nucleations of datapoints. For example, straight lines suggest straight paths, a line that changes slope insinuates refraction, and power relations indicate reflections and out-of-plane paths.

Inversion - Tomographic Images

The tomographic data for the two cases are inverted using matrix-based algorithms. Decisions related to the inversion procedure are made on the bases of the information obtained during pre-processing.

Summary of Pre-processing Results - Initial Guess

Pre-processing results obtained for the balloon tomographic data indicate that resolution is constrained by ill-posedness which increases for a resolution greater than 8x8, unless additional information is added. The host medium is homogeneous and isotropic with velocity V_{host}≈343 m/s. There is a high velocity anomaly; the graphical back-projection of its shadows suggests that the anomaly is centrally located. In summary, the velocity of the host medium is used as the initial guess: $(s_j)_o=(V_j)^{-1}{}_o$ =(343 m/s), the medium is discretized in 10x10 pixels, and image enhancement will be guided to highlight contrast with a high velocity anomaly.

Preprocessing results for the tunnel data indicate that resolution is constrained by wavelength λ=3m rather than by the separation between transducers (0.2m) or the density of the spatial coverage. The medium is anisotropic and vertically heterogeneous. There is at least one low velocity region suggested by the shadows. The selected initial guess for the slowness field $(s_j)_o=(V_j)^{-1}{}_o$ $=(3160 + 12\cdot z)^{-1}$, and the medium is discretized in 10x30.

Inversion Procedure

Matrix inversion methods are versatile. However, since the matrix of travel lengths $\underline{\underline{L}}$ is large, matrix-based inversion is storage and computer demanding. In a dense n×n matrix, the order of computation complexity is $O(n^3)$, and $O(n^2)$ for storage. On the other hand, the matrix $\underline{\underline{L}}$ is sparse. Even though there are n pixels (columns in $\underline{\underline{L}}$), only few pixels are touched by a given ray. For example, consider the tunnel tomogram: there are n=10x30=300 pixels, yet only 10-to-15 pixels are touched by any cross-hole ray. Thus there are only 10 to 15 non-zero entries, out of the 300 elements in each row of $\underline{\underline{L}}$. There are efficient algorithms to process sparse matrices, which can reduce the order of computational complexity to $O(n^{1.3})$ and storage requirements to O(n) (Press, et al. 1992). The tomographic software written as part of this study uses sparse matrix algorithms (Gheshlaghi et al*.* 1995).

The tomographic data gathered in the two case histories were inverted using the regularized least squares procedure with initial guess (Tarantola 1987; Santamarina and Gheshlaghi 1995),

$$\underline{s} = \underline{s}_o + \left[\left(\underline{\underline{L}}^T \underline{\underline{L}} + \lambda \cdot \underline{\underline{R}}^T \underline{\underline{R}} \right)^{-1} \cdot \underline{\underline{L}}^T \cdot \left(t - \underline{\underline{L}} \cdot \underline{s}_o \right) \right] \qquad (8)$$

The regularization matrix was based on laplacian smoothing. The smoothing kernel for a central pixel is [(0,1,0),(1,-4,1),(0,1,0)]. Kernels for boundary pixels were developed following the reflection rule. The optimal regularization coefficient λ was determined from the maximum value of the joint distribution between computed and measured travel times (Gheshlaghi 1997; Menke 1989).

Post-processing

The computed tomograms are presented in Figure 8. Pixel values were thresholded to enhance contrast. Notice the ghost in the upper part of the tunnel tomogram which results from the low information density in that region. The smearing of the tunnel in the direction of the rays is a consequence of limited illumination angles.

Conclusions

Measurement and model errors are magnified during data inversion in ill-posed inverse problems. Data pre-processing can help

- identify measurement errors,
- recognize the general characteristics of the host medium and the presence of anomalies,
- define a good initial guess of the solution which is used to anchor the inversion,
- select adequate models (e.g., straight or curved rays, isotropic or anisotropic media),
- chose the proper resolution for the tomogram, taking into consideration wave physics, data density, and information content,
- corroborate that the resulting image is not an artifact of the inversion procedure that could result from the uneven spatial coverage and the presence of model and measurement errors.

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