EMERGENT PHENOMENA IN SPATIALLY VARYING SOILS

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Abstract

Spatial variability in soils gives rise to various phenomena that do not take place in homogeneous media. We report in this document emergent phenomena related to variability in stiffness, strength, conduction and diffusion. Stiffness variability triggers stress focusing along stiffer percolating zones, and alters elastic wave propagation causing ray bending, mode conversion, diffraction healing, and coda. The heterogeneous distribution of strength promotes localized shear failure along interconnected weaknesses. Excess pore water pressure generation and dissipation are highly sensitive to the variability in stiffness and hydraulic conductivity. The effective hydraulic conductivity decreases with increasing variability in *k*-fields. Spatial correlation plays a secondary role in the absence of high-*k* percolating paths or low-*k* transverse seams. Most analytical closed-form solutions for effective stiffness, strength, conduction or diffusion are based on volume fractions and fail to capture the spatial distribution and correlation that are inherent in natural sediments.

Keywords: spatial variability, stiffness, strength, conduction, diffusion, consolidation

Introduction

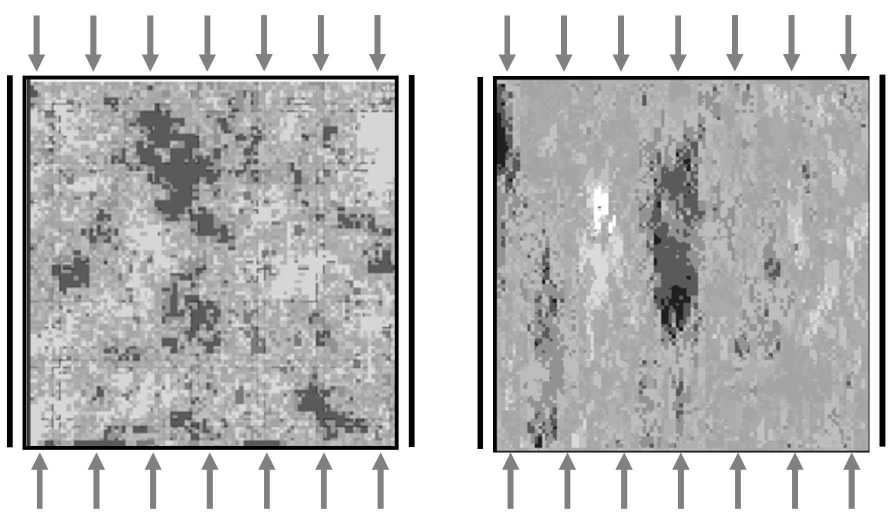
Spatial heterogeneity is an inherent characteristic in soils. Spatial variability can be described using statistical parameters such as the mean μ, the coefficient of variation *COV*, and the correlation length (Vanmarke 1977; Asaoka and A-Grivas 1982; DeGroot and Baecher 1993; Lacasse and Nadim 1996; Phoon and Kulhawy 1999).

The role of spatial variability in geotechnical engineering problems have been explored in previous studies, including: deformation (Baecher and Ingra 1981; Zeitoun and Baker 1992; Paice et al. 1996), strength (Popescu et al. 1996; Griffiths and Fenton 2001), conduction (Renard and de Marsily 1997; Wen and Gómez-Hernández 1996), and diffusion (Schiffman and Gibson 1964; Nishimura et al. 2002).

Spatial heterogeneity brings about new phenomena that do not take place in homogeneous media. In this study, we explore the effects of spatial variability in soils, and place emphasis on the identification of emergent phenomena. The methodology is based on numerical simulations (ABAQUS and MatLab). The matrix decomposition technique is used to realize multidimensional correlated random fields that exhibit preselected values of the mean, standard deviation and correlation length of the soil parameters of interest (details in El-Kadi and Williams, 2000). The complete Monte-Carlo simulations and results can be found in Kim (2005) and Narsilio (2006).

Stiffness in Spatially Varying Media

A heterogeneous medium with spatially varying stiffness is subjected to zero-lateral strain loading (Figure 1a). Spatial heterogeneity is applied on the small-strain shear modulus, within the framework of the modified Duncan-Chang model, assuming $G_0=\alpha(\sigma')^{\beta}$ (Duncan and Chang 1970). The relative correlation length is 10% of the domain size. Figure 1b shows that the vertical load transfer concentrates along the percolating stiffer parts of the medium (lighter-colored regions in Figure 1a). Stress focusing leads to lower global effective stiffness and smaller horizontal K_0 load transfer with higher variability.



(a) G_0 distribution and boundary conditions (b) σ_v' distribution

Figure 1. Initial heterogeneity in stiffness and corresponding vertical stress distribution under zero-lateral strain loading. Lighter colors indicate higher values.

Elastic Wave Propagation in Spatially Varying Media

Elastic wave propagation is simulated through media with different types of variability: a homogeneous medium (Figure 2a), a vertically heterogeneous medium (Figure 2b), an uncorrelated heterogeneous medium (Figure 2c), and a correlated heterogeneous medium (Figure 2d). The arithmetic mean shear wave velocity is the same in all cases $\mu[V_s]$. A vertical impulse-type excitation is applied at the middle of the left boundary. Snapshots of the particle motion fields are shown at selected times T=0.25T_0, 0.50T_0, 0.75T_0 and T_0, where T_0 is the side-to-side shear wave travel time for the homogeneous medium with the velocity $\mu[V_s]$, i.e. $T_0=L/\mu[V_s]$.

Stress-induced vertical stiffness heterogeneity is inherent in soils. Vertical heterogeneity produces ray bending according to Fermat's minimum travel time principle (Figure 2b). When the wavelength approaches the scale of the correlation length, wave signatures are complex, reflect ballistic-type propagation, experience low-pass filtering, and signatures have long codas (Figures 2c and 2d).

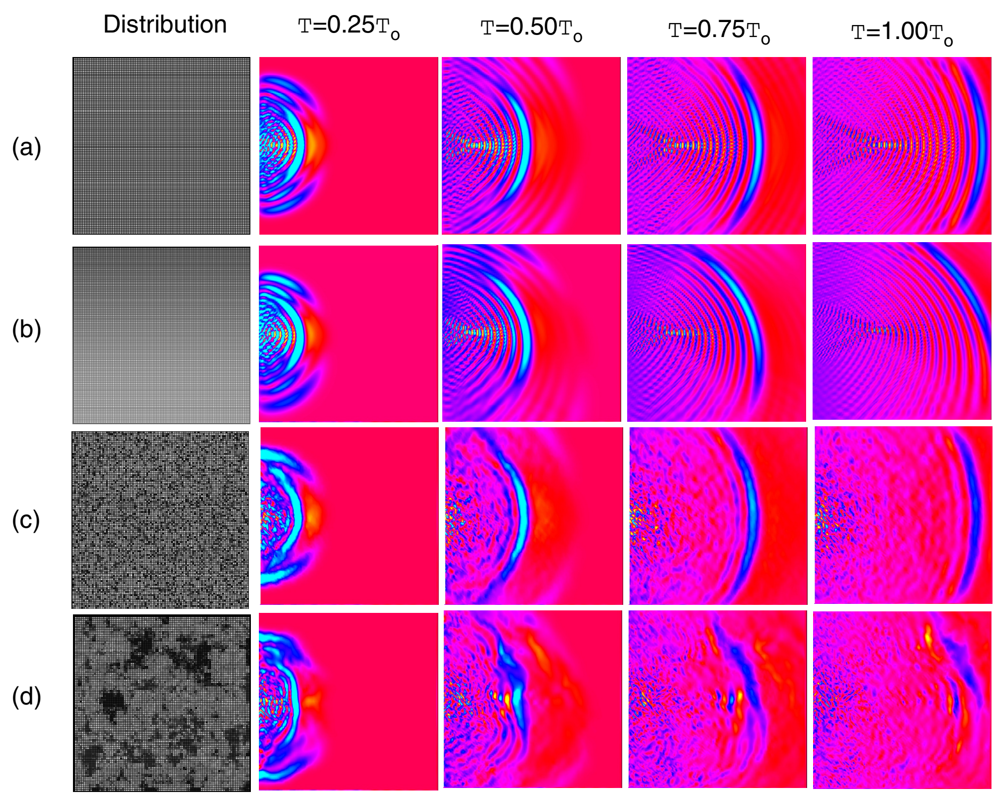
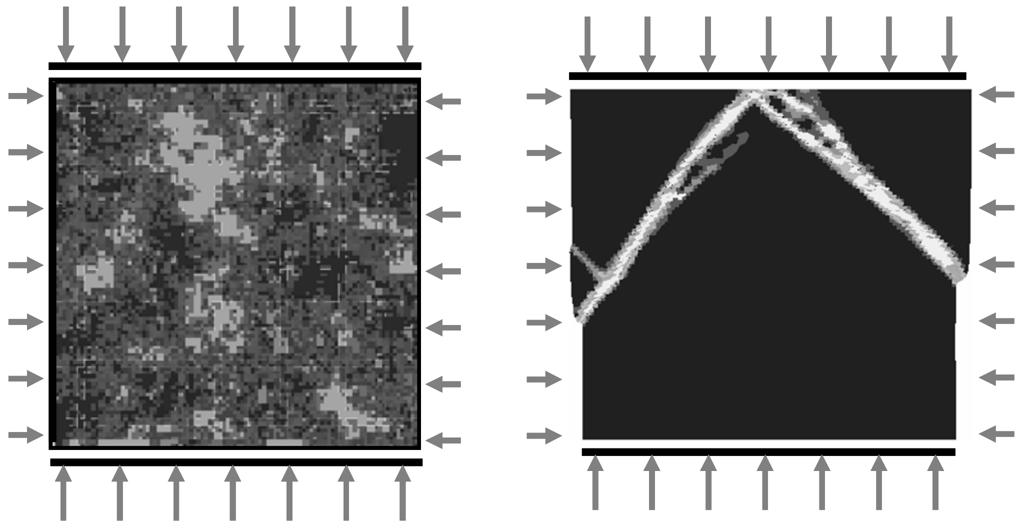


Figure 2. Elastic propagation though heterogeneous elastic media. (a) Homogeneous medium, (b) vertically varying medium, (c) uncorrelated random medium, (d) correlated random medium. Lighter colors indicate higher values.

Strength in spatially varying media

The undrained deviatoric load response is studied using strain-controlled deviatoric loading simulations without local or global drainage. The medium has spatially varying undrained shear strength captured by spatially varying void ratio e_0 in a modified Cam-clay model (Roscoe and Burland 1968; Figure 3a). The shear strain distribution after 5% nominal axial strain exhibits clear strain localization along the interconnected weak elements (Figure 3b).



(a) e_0 distribution and boundary conditions (b) shear strain distribution

Figure 3. Initial heterogeneity in void ratio and corresponding shear strain distribution under undrained deviatoric loading. Lighter colors indicate higher values.

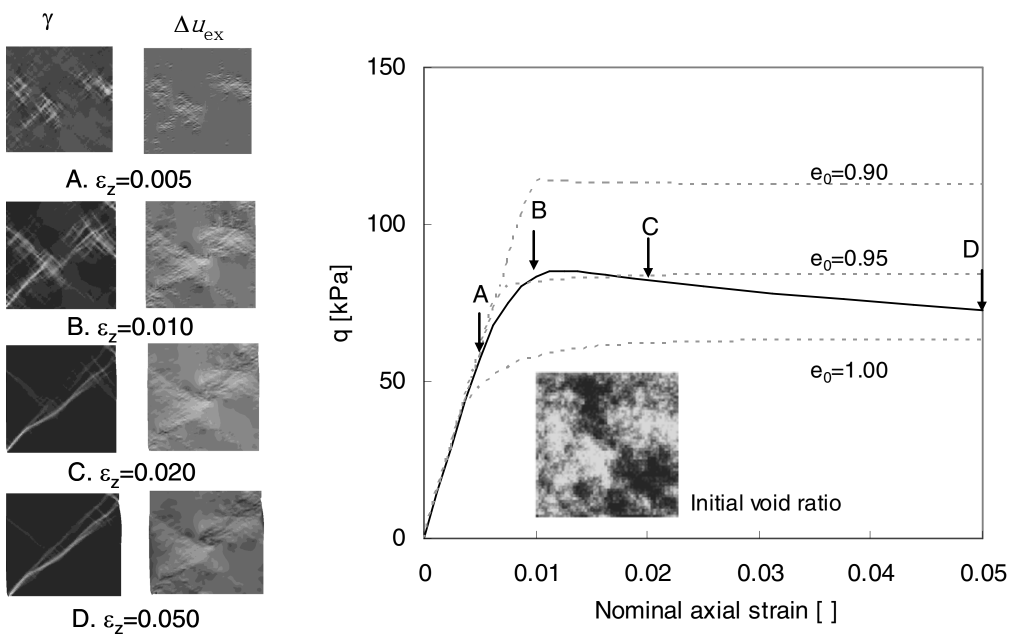
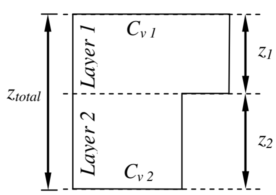


Figure 4. Internal shear strain γ evolution and corresponding excess pore pressure Δu_{ex} development in a heterogeneous medium subjected to undrained deviatoric loading. Lighter colors indicate higher shear strain, pore pressure or void ratio. Uniformly distributed initial void ratio range $e_0 = 0.8$ to 1.0. The correlation length is 10% of the mesh size. Note: $e_{cs} = 0.92$ for $\sigma_0' = 100$ kPa. The dotted lines are for homogeneous media at the indicated void ratios.

The development of excess pore water pressure is associated to the level of shear strain (Figure 4). The equivalent undrained shear strength decreases as the variability in e_0 increases (not shown here – details and complete results can be found in Kim 2005).

Diffusion in Spatially Varying Media

Diffusion is the time-dependent spatial evolution of a parameter towards its steady state condition. In particular, consolidation is the diffusion of excess pore pressure u_e, and it depends on the hydraulic conductivity k and the skeletal compressibility m_v of soils as captured in the diffusion coefficient, i.e., the coefficient of consolidation $C_v=k/(\gamma_w \cdot m_v)$ where γ_w is the unit weight of water. The variation in C_v with depth must be accounted for to properly explain the observed field response in many case histories (Abbot 1960; Nishimura et al. 2002). Most situations require numerical analysis (Papanicolaou and Diplas 1998; Schiffman and Gibson 1964; Yang et al. 2004).



$$\alpha = \frac{z_1}{z_1 + z_2} \qquad \beta = \frac{C_{v1}}{C_{v2}} \qquad T = \frac{C_{v2} \cdot t}{H^2}$$

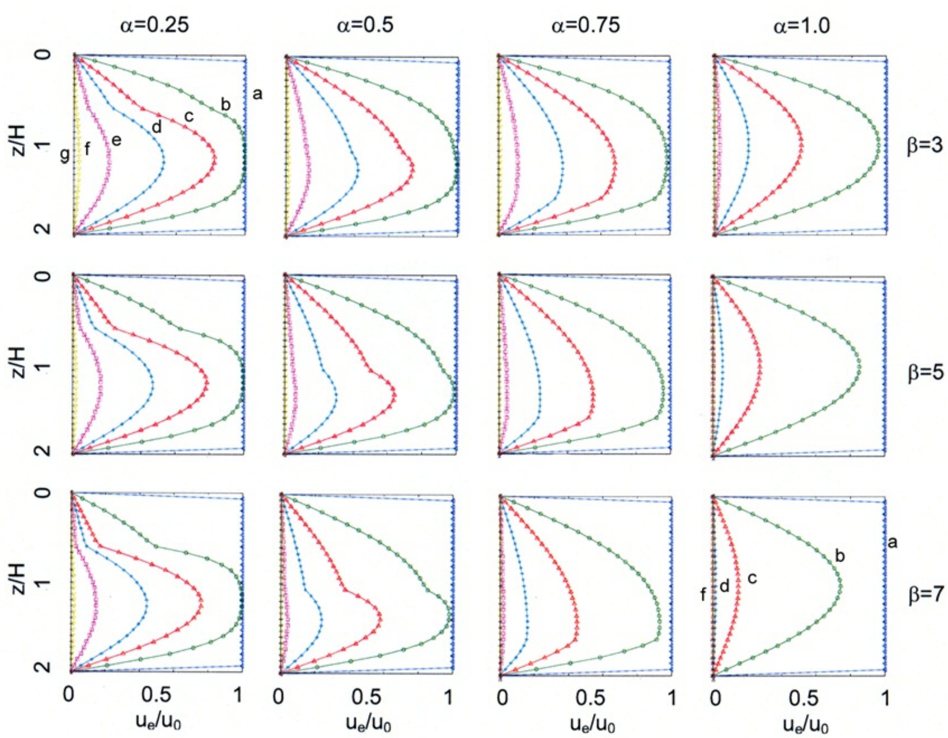


Figure 5. Excess pore pressure charts for bi-layer systems. Notice that α=0 or α=1 corresponds to the homogeneous case. Isochrones at: (a) T=0, (b) T=0.03125, (c) T=0.125, (d) T=0.25, (e) T=0.50, (f) T=1.0, and (g) T=2.0.

In this study, the continuous diffusion problem is written in discrete form and processed using the construct of matrices and vectors under the finite difference Crank-Nicholson scheme. Figure 5 shows the normalized excess pore water pressure u_e/u_0 profiles with depth at selected dimensionless times for various geometric α and diffusion β ratios, for a constant initial excess pore pressure with depth u_0, and double drainage conditions. The dissipation in the higher C_v layer (the top layer) takes place much faster as β increases, yet, the total dissipation is controlled by lower C_v and thickness ratio α. The gradient at the interface is $\partial u/\partial z \neq 0$; therefore, water flow takes place across the interface for all α and β values, indicating interaction between layers (e.g. water-rich layer development; see Kokusho 1999).

Conduction in Spatially Varying Media

The presence of fluids affects all aspects of soil behavior, including chemical, mechanical, and biological processes. The general form of Laplace's equation is obtained by combining Darcy's law in the three directions x, y and z (with hydraulic conductivities k_x, k_y and k_z), Bernoulli's energy equation, and the change in volume in soils as a function of the degree of saturation S and void ratio e (Richards 1931),

$$k_x \cdot \frac{\partial^2 h}{\partial x^2} + k_y \cdot \frac{\partial^2 h}{\partial y^2} + k_z \cdot \frac{\partial^2 h}{\partial z^2} = \frac{1}{1+e} \cdot \left(e \cdot \frac{\partial S}{\partial t} + S \cdot \frac{\partial e}{\partial t} \right)$$

where h is the total head. It follows from this equation that fluid flow is affected by the spatial variability of the hydraulic conductivity (Renard and de Marsily 1997; Wen and Gómez-Hernández 1996).

Numerical simulations of spatial variability of hydraulic conductivity are performed using finite element modeling and a Matlab code is written for this purpose. Cases in 2D are captured using the Garlekin method to solve the Laplace equation with prescribed total head and flow boundary conditions (Dirichlet and Neumann boundary conditions). The code is validated against closed-form solutions for simple geometries. Once the Laplacian is solved, the equivalent hydraulic conductivity is computed by invoking Darcy's law to represent a non-homogeneous medium by means of a homogeneous medium that allows equal flow through (Bøe 1994; Cardwell and Parsons 1945; Warren and Price 1961). If q_{el} is the flow through each element on a given equipotential line (e.g., either inlet or outlet surfaces), A is the area of that surface and $\Delta h/L$ is the imposed hydraulic gradient, then the equivalent hydraulic conductivity k_{eq} is computed as:

$$k_{eq} = \frac{L}{\Delta h \cdot A} \int_A q_{eq} \, dA$$

Multiple realizations of correlated random hydraulic conductivity 2D fields are generated with a correlation length equal to 20% of the mesh size. Figure 6 shows typical results for the spatial distribution of total head and flow lines (the complete study is documented in Narsilio, 2006). The variation of the computed equivalent k versus COV in log-normal distributions of hydraulic conductivity for both correlated and uncorrelated random fields is shown in Figure 7. Contrary to one's a-priori intuition, these results show that increased variability in k (for the same mean value $\mu[k]$) lead to lower equivalent conductivity. Moreover, there is a tendency to higher equivalent hydraulic conductivity for uncorrelated fields than in correlated *k*-fields. These results indicate that (in the absence of high-*k* percolating paths), the effect of low-*k* regions prevails over high-*k* zones, i.e., more of a series rather than a parallel system. Thus, variability brings about tortuosity and a harmonic mean averaging along flow paths.

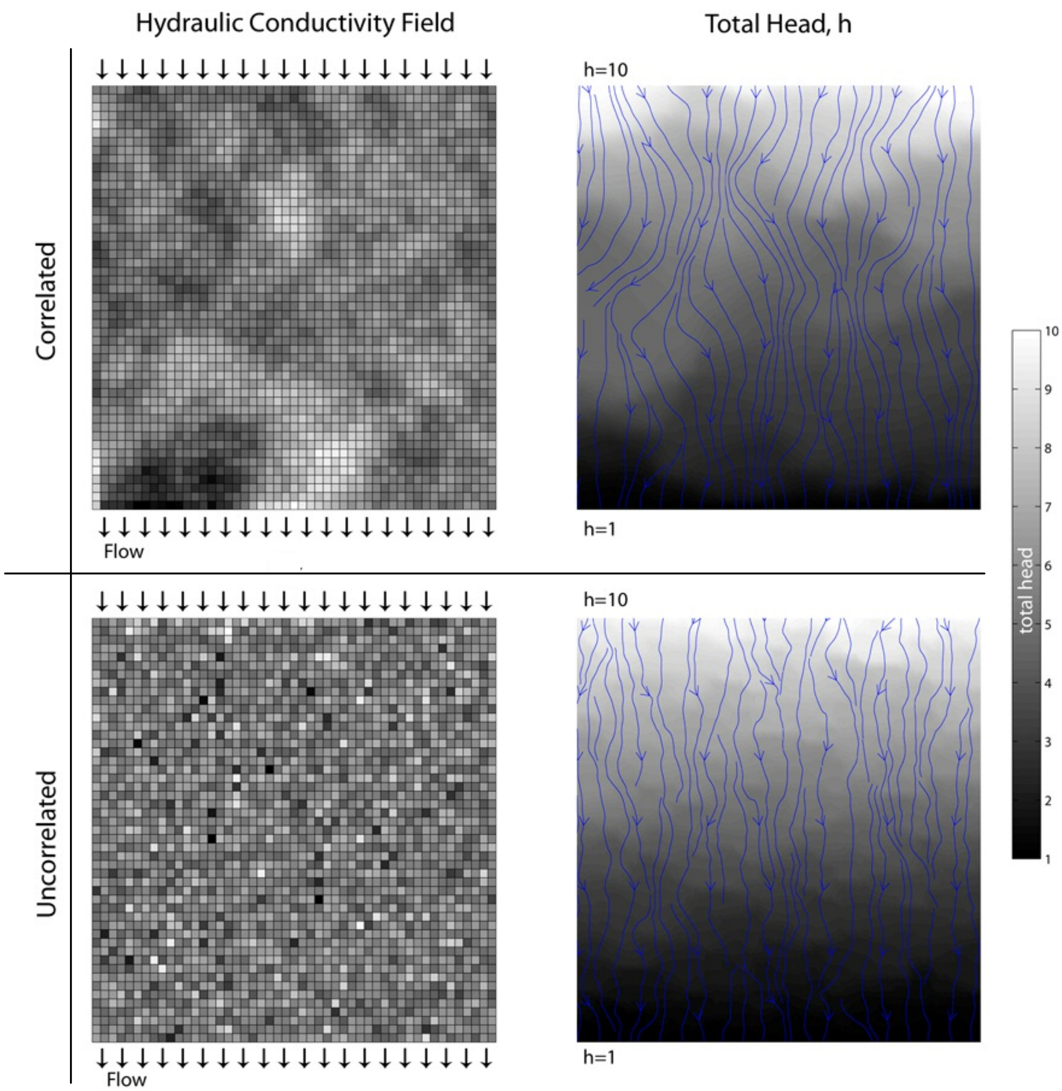


Figure 6. Spatial variability in hydraulic conductivity – Correlated and uncorrelated random *k*-fields and associated total head and flow lines.

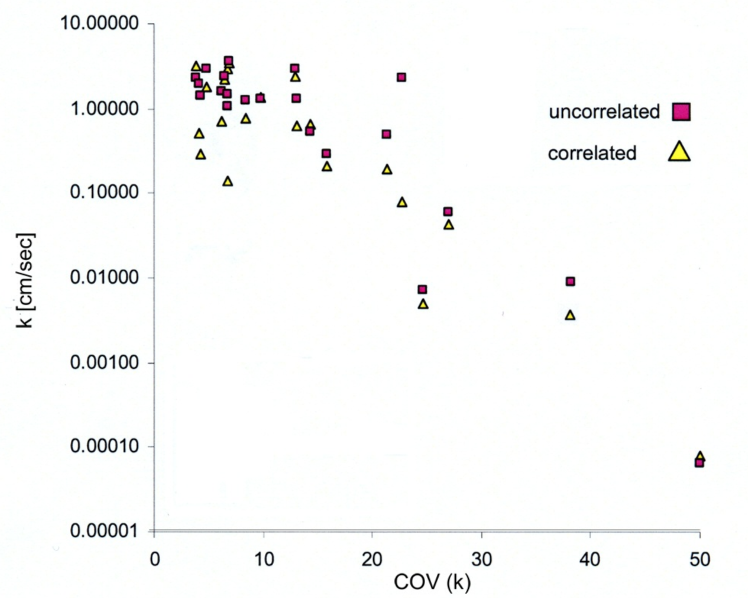


Figure 7. Equivalent hydraulic conductivity as a function of variability in *k*. The correlation length is 20% of the mesh size.

Conclusions

Spatial variability affects the stiffness, strength, diffusion, and conduction properties of soils, and causes new emergent phenomena that are not present in homogeneous continuous media. Results from a comprehensive study (partially presented here) show:

- Load transfer in zero-lateral strain loading concentrates along stiffer zones, leading to "stress-focusing" and lower K_0 horizontal load transfer.
- Spatial variability in elastic modulus alters elastic wave propagation yielding ray bending, mode conversion, diffraction healing, and coda.
- The local evolution of excess pore pressure during locally and globally undrained deviatoric loading correlates with the strain localization that develops along interconnected weakness.
- The dissipation of excess pore water pressure is intimately related to the layers C_v, thickness, and spatial arrangement. Water flow across the interface between layers induces interaction between the layers.
- The equivalent hydraulic conductivity decreases with increasing variability for the same mean value $\mu[k]$.
- Correlated random *k*-fields exhibit lower equivalent hydraulic conductivity than uncorrelated *k*-fields in spite of identical statistics in global hydraulic conductivity.

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