Soil Response to Repetitive Changes in Pore-Water Pressure under Deviatoric Loading

Junghhee Park¹ and J. Carlos Santamarina, A.M.ASCE²

Abstract: Soils often experience repetitive changes in pore water pressure. This study explores the volumetric and shear response of contractive and dilative sand specimens subjected to repetitive changes in pore water pressure, under constant deviatoric stress in a triaxial cell. The evolution towards a terminal void ratio \( e_T \) characterizes the volumetric response. The terminal void ratio \( e_T \) for pressure cycles falls below the critical state line, between \( e_{\text{min}} < e_T < e_{cs} \). Very dense specimens only dilate if they reach high stress obliquity \( \eta_{\text{max}} \) during pressurization. The terminal void ratios for very dense and medium dense specimens do not converge to a single trend. The shear deformation may stabilize at shakedown, or continue in ratcheting mode. The maximum stress obliquity \( \eta_{\text{max}} \) is the best predictor of the asymptotic state; shakedown prevails in all specimens subjected to stress obliquity \( \eta_{\text{max}} < 0.95 \cdot \eta_{cs} \) and ratcheting takes place when the maximum stress obliquity approaches or exceeds \( \eta_{\text{max}} \geq 0.95 \cdot \eta_{cs} \). Volumetric and shear strains can accumulate when the strain level during pressure cycles exceeds the volumetric threshold strain (about \( 5 \times 10^{-4} \) in this study). A particle-level analysis of contact loss and published experimental data show that the threshold strain increases with confinement \( p_{\text{w}}' \). DOI: 10.1061/(ASCE)GT.1943-5606.0002229. This work is made available under the terms of the Creative Commons Attribution 4.0 International license, http://creativecommons.org/licenses/by/4.0/.

Author keywords: Ratcheting; Pore water pressure cycle; Shakedown; Stress obliquity; Terminal void ratio.

Introduction

Soils experience repetitive changes in pore water pressure during groundwater level oscillations associated with tidal and river level fluctuations, and engineered structures such as docks and managed reservoirs (O’Reilly and Brown 1991; Chu et al. 2003; Orense et al. 2004; Leroueil et al. 2009; Page et al. 2010; Nakata et al. 2013; Shi et al. 2016). Coupled processes may also cause pore-fluid pressure oscillations, for example, in the case of a soft clay subjected to temperature cycles (Abuel-Naga et al. 2007).

Pore-fluid pressure fluctuations affect a wide range of geotechnical systems from foundations and slope stability to pumped-storage hydroelectric power stations, aquifer storage and recovery systems, compressed air energy storage, enhanced oil recovery by cyclic water flooding and cyclic steam injection, and repetitive CO₂ injection (Premchitt et al. 1986; Olson et al. 2000; Gambolati and Teatini 2015; Huang 2016; Chang et al. 2017).

Soils gradually deform in response to all kinds of repetitive excitations. Repetitive changes in water pressure imply effective stress cycles that can lead to the accumulation of plastic volumetric and shear strains. This study explores the volumetric and shear response of contractive and dilative sands subjected to repetitive changes in pore water pressure under constant deviatoric stress. The following section presents a detailed review of the state of the art and identifies salient gaps in knowledge.

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Shear Asymptotic State: Shakedown or Ratcheting?

The asymptotic condition in shear governs the response of all structures, from pavements (Sharp and Booker 1984) to metals (Johnson 1986). The shear response falls into one of three asymptotic regimes, as observed on the \(q-\gamma\) quadrant in Fig. 1 (Alonso-Marroquin and Herrmann 2004; Werkmeister et al. 2005)

- Elastic shakedown: non-hysteretic, fully recoverable deformation in every cycle.
- Plastic shakedown: hysteretic stress-strain response without permanent deformation at the end of each cycle.
- Ratcheting: the stress strain response is hysteretic, and there is continued plastic strain accumulation in every cycle.

The asymptotic condition depends on the stress amplitude ratio, the cyclic stress ratio \(\Delta\sigma_{\text{amp}}^z / 2p_o^0\), and the cyclic shear stress level \(\Delta\tau_{\text{amp}} \sigma^z / \Delta\sigma_{\text{amp}}^z\) (Wu et al. 2017; Cai et al. 2018; Gu et al. 2018). Ratcheting should be expected at large stress amplitudes and high stress obliquity \(\eta = q / p^0\). It may also develop when a large number of cycles reaches a fatigue-induced tipping point, or when the stress level causes particle crushing (see data in Werkmeister 2003; Alonso-Marroquin and Herrmann 2004; Werkmeister et al. 2005; Wichtmann et al. 2005; da Fonseca et al. 2013).

Experimental Study

Tested Sand: Properties

Table 1 summarizes the main characteristics of the “KAUST 20/30 sand” used throughout this study and includes index properties such as the particle shape, the coefficient of uniformity \(C_u\), and the extreme void ratios \(e_{\text{max}}\) and \(e_{\text{min}}\). Measured values are compared against predicted values from index properties for self-consistent verification (refer to Table 1 for details).

The critical state provides a “reference asymptotic state” for this study. Fig. 2 shows data for a set of conventional consolidated-drained CU triaxial tests projected onto \(p'-q-e_\gamma-e-u\) planes.

![Fig. 1. (Color) Anticipated soil response to pore water pressure cycles under constant deviatoric loading. The plot captures asymptotic conditions in shear strain (shakedown or ratcheting) and volumetric strain (terminal void ratio \(e_T\).)](image)

### Table 1. Tested sand—Properties

<table>
<thead>
<tr>
<th>Property</th>
<th>KAUST 20/30 sand</th>
<th>Observations (Verifications using index test data)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Particle diameter</td>
<td>(d = 0.60 \sim 0.85)</td>
<td>Image analysis—Roundness (R = \sum r_i / N). The average radius of curvature of surface features divided by the radius of the largest inscribed sphere (r_{\text{max}})</td>
</tr>
<tr>
<td>Roundness</td>
<td>(R = 0.60)</td>
<td>Estimated maximum void ratio: (e_{\text{max}} = 0.76) (Youd 1973)</td>
</tr>
<tr>
<td>Coefficient of uniformity</td>
<td>(C_u = 1.20)</td>
<td>Estimated minimum void ratio: (e_{\text{min}} = 0.54) (Cho et al. 2006)</td>
</tr>
<tr>
<td>Specific gravity</td>
<td>(G_s = 2.65)</td>
<td>Angle of repose method: (\phi_{\text{c}} = 32^\circ) (Santamarina and Cho 2001)</td>
</tr>
<tr>
<td>Maximum void ratio</td>
<td>(e_{\text{max}} = 0.786)</td>
<td>Interfered from roundness is (\phi_{\text{c}} = 31^\circ \pm 2^\circ) (Cho et al. 2006)</td>
</tr>
<tr>
<td>Minimum void ratio</td>
<td>(e_{\text{min}} = 0.533)</td>
<td>Intercept of CSL at 1 kPa</td>
</tr>
<tr>
<td>Friction angle at constant volume shear</td>
<td>(\phi_{\text{c}} = 31^\circ)</td>
<td>Slope of CSL</td>
</tr>
<tr>
<td>Critical state line</td>
<td>(\Gamma = 0.845)</td>
<td>Coefficients of CSL</td>
</tr>
<tr>
<td>CSL in (\log p')</td>
<td>(\lambda = 0.074)</td>
<td>Corresponds to Hertzian-based power model (V = \alpha \cdot (p'_j / kPa)^{\beta})</td>
</tr>
<tr>
<td>Shear wave velocity parameters</td>
<td>(\alpha = 89) m/s</td>
<td>Note: (\alpha) and (\beta) values for (e_o = 0.65)</td>
</tr>
<tr>
<td></td>
<td>(\beta = 0.21)</td>
<td>Based on contact-loss analysis: (\gamma_{\text{loss}} = 1.3 \cdot (\sigma'_j / G_s)^{2/3})</td>
</tr>
<tr>
<td>Estimated threshold strain for contact</td>
<td>(\gamma_{\text{th}}</td>
<td>_{\text{loss}} = 5 \times 10^{-4})</td>
</tr>
<tr>
<td>loss in monotonic loading</td>
<td></td>
<td>Mineral: (G_s = 30) GPa and (v = 0.25) (assumed in analysis)</td>
</tr>
</tbody>
</table>

Note: Measured values are compared against values predicted from index properties for self-consistent verification.
We prepare loose, medium dense, and dense specimens using a combination of raining and tamping techniques to obtain different initial relative densities between $D_r = 15\%$ and $70\%$. Enables cyclic changes in pore water pressure and measures volume changes.

**Experimental Devices and Configuration**

The triaxial system used to conduct the repetitive pressure cycles consists of (1) a triaxial cell with an LVDT (Linear Variable Differential Transformer) to track the vertical displacement, (2) a loading frame to apply a constant deviatoric stress, and (3) a pressure panel that generates cyclic changes in pore water pressure and measures volume changes.

**Sample Preparation**

We prepare loose, medium dense, and dense specimens using a combination of raining and tamping techniques to obtain different initial relative densities between $D_r = 15\%$ and $70\%$.

**Loading Histories**

We can simulate the effects of changes in water pressure through changes in either the back pressure or the confining pressure (Brenner et al. 1985; Anderson and Sitar 1995; Farooq et al. 2004; Orense et al. 2004). The two test procedures yield the same results if the Biot’s coefficient $\chi = 1 - B_{st}/B_g$ remains close to $\chi \approx 1.0$, that is at a relatively low confining effective stress (note: $B_{st}$ is the bulk modulus of the soil skeleton, and $B_g$ is the bulk modulus of the mineral that makes the grains, Skempton 1961; Santamarina et al. 2001). In this study, we control the back pressure $u_b$.

Fig. 3 presents a subset of the stress paths explored in this study. Typical loading histories consist of five stages (1) isotropic consolidation, (2) drained deviatoric loading to stress obliquity $\eta = 0.33$, (3) a decrease in back pressure $u_b$ to reach $\eta = 0.20$, (4) repetitive changes in pore water pressure from $\eta = 0.20$ to $\eta_{\text{max}} = 0.50$ for $N = 100$ loading cycles (shown in red), and (5) strain-controlled undrained axial compression from $\eta = 0.20$ to failure at a vertical strain rate of $\varepsilon_z = 0.01/\text{min}$. Table 2 summarizes the experimental study. Test parameters include the initial void ratio $\varepsilon_0$, cyclic pressure amplitude $\Delta u_w$, and maximum stress obliquity $\eta_{\text{max}} = q/p_{\text{min}}$. Cyclic pressure amplitudes $\Delta u_w$ selected for this study represent various field conditions, such as tidal action (<170 kPa at Burntcoat Head and Leaf Basin in North America), seasonal fluctuations in ground water levels (70–100 kPa.—Hung et al. 2012; Huang 2016), injection pressures for injection wells and injection-recovery wells used in aquifer storage (<500 kPa.—Shi et al. 2016; Page et al. 2010), and some coupled processes (e.g., geothermally-induced $\Delta u_w < 250$ kPa.—Laloui 2001; Abuel-Naga et al. 2007).

**Experimental Results**

This section reports detailed experimental results for two sets of tests designed to explore the effects of maximum stress obliquity $\eta_{\text{max}}$ and initial confinement $p_{\text{min}}'$. We analyze the complete dataset gathered in this study in the following section.
undrained shear compression from ratio and the vertical strain accumulation during the repetitive
consists of five stages: (1) isotropic consolidation, (2) drained deviatoric loading to stress obliquity
Repetitive pressure cycles
medium dense sands initially loaded to the same
Fig. 3. Pre-loading
Fig. 4. The void ratio decreases during isotropic confinement (p′ = 100 kPa, q = 0) and deviatoric loading (p′ = 150 kPa, q = 50 kPa). The vertical strain is very similar in all specimens as the stress obliquity reaches the initial value of γp = 0.33 and during the first decrease in pore water pressure to reach \( \eta_{\text{max}} = 0.20 \).

\( \eta \rightarrow \eta_{\text{min}} \) and accumulates at the end of every cycle. Volumetric contraction and vertical strain accumulation are more pronounced in specimens that reach a higher maximum stress obliquity \( \eta_{\text{max}} \) during pressure cycles. Note that the initial void ratio \( \epsilon_0 \) of all specimens falls in the contractive zone just before repetitive loading; thereafter, the two specimens subjected to large pressure cycles (\( \eta_{\text{max}} = 0.50 \) and \( \eta_{\text{max}} = 0.45 \)) become denser than at the critical state.

Undrained shear. All specimens reach the critical state line during the undrained deviatoric loading that followed the \( N = 100 \) pressure cycles (\( p'-q-e \) space in Fig. 4). These results confirm that in the absence of overt localization the critical state line is not affected by the monotonic or cyclic loading history (Taylor 1948; Schofield and Wroth 1968; Castro et al. 1982; Mohamad and Dobry 1986).

Study 2: Confining Effective Stress \( p' \)
Fig. 5 shows the \( p'-q-e-\epsilon_2 \) load-deformation response of three medium dense specimens subjected to different initial mean stress values \( p'_0 \). Initial conditions include specimens above and below the critical state line. Details of the loading history before repetitive loading is shown in Fig. 3. Pressure cycles cause changes in obliquity from \( \eta_{\text{min}} = 0.20 \) to \( \eta_{\text{max}} = 0.50 \) in all cases. The changes in void ratio and the vertical strain accumulation during the repetitive pressure cycles are more significant in the one specimen subjected to high initial mean stress \( p'_0 \). Once again, all specimens shear and dilate as the pressure increases. However the overall void ratio trend is contractive at the end of every cycle. All three specimens land on the dilative side of the critical state at the end of cyclic loading and exhibit a dilative tendency during the final undrained shear.

Analysis of the Complete Dataset
This section analyzes the results of all tests conducted in this study (Table 2), with an emphasis on the shear strains and volume changes that occur during repetitive pressure cycles. Within triaxial boundary conditions, the shear strain \( \gamma = (3\epsilon_2 - 2\epsilon_3)/2 \) combines the vertical strain \( \epsilon_2 \) and the volumetric strain \( \epsilon_{\text{vol}} \). System compliance and inadequate saturation bias both the measured peak-to-peak volumetric strain and the computed peak-to-peak shear strain. Therefore, figures and analyses in this section place emphasis on incremental and cumulative strains determined at the same pressure at the end of each cycle.

Shear Deformation
Fig. 6 presents the shear strain accumulation as a function of pressure cycles. The initial mean stress is the same for all specimens, \( p'_0 = 250 \) kPa, but pressure cycles reach different maximum stress obliquities \( \eta_{\text{max}} \). The shear strain accumulation model below fits data trends in all tests (modified from Chong and Santamarina 2016)

\[
\gamma_i = \gamma_1 + a(1 - i^{-b}) - c(1 - i^{-d}) + d(i - 1)
\]

where \( a, b, c, \) and \( d \) are fitting parameters, and \( i \) is the number of loading cycles. The shakedown response corresponds to \( d = 0 \), while \( d > 0 \) implies ratcheting. Table 2 summarizes the fitted model parameters for all tests. Results indicate that

- The shear strain accumulation induced by pressure cycles is more pronounced in earlier cycles, in loose sands; in specimens that experience a higher maximum stress obliquity \( \eta_{\text{max}} \) (for tests with the same initial \( p'_0 \)), and in specimens subjected to a higher initial mean stress \( p'_0 \) (for tests that reach the same \( \eta_{\text{max}} \)).

- Shakedown is unmistakable for specimens with small \( \eta_{\text{max}} \). In general, all specimens subjected to stress obliquity \( \eta_{\text{max}} \leq 0.50 \).
<table>
<thead>
<tr>
<th>Specimen characteristics</th>
<th>Stress conditions</th>
<th>Pressure cycles</th>
<th>Shear strain $\gamma^a$</th>
<th>CU—AC</th>
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</thead>
<tbody>
<tr>
<td>Conditions before</td>
<td>$q$ [kPa]</td>
<td>$\Delta u_w$ [kPa]</td>
<td>$p_{\text{max}}'$ [kPa]</td>
<td>$e_0$</td>
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<td>pressure cycles</td>
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<td>relative to critical</td>
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<td>state</td>
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<td>Test No.</td>
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<td>B-value</td>
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<td>$e_T$</td>
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<td>$\gamma_i$</td>
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</table>

Note: Fitting parameters correspond to models introduced in the text. $q$ = deviatoric stress; $\Delta u_w$ = cyclic pressure amplitude; $p_{\text{max}}'$ = maximum mean stress at the end of depressurization cycle; $\eta_{\text{max}}$ = maximum stress obliquity ($= q/p_{\text{max}}'$); $e_0$ = initial void ratio at $i = 0$; $e_T$ = terminal void ratio at $i \to \infty$; $m$ = model parameter; $N^c$ = characteristic number; $\gamma_i$ = shear strain at the end of first cycle $i = 1$; $a$, $b$, and $c$ = model parameters; and $d$ = ratcheting parameter.

Shear strain accumulation model: $\gamma_i = \gamma_1 + a(1 - i^{-b}) - c(1 - i^{-d}) + d(i - 1)$.

Void ratio evolution model: $e_i = e_T + (e_0 - e_T)[1 + (i/N^c)^a]^{-1}$. 

Related Keywords: shear strain, shear stress, cyclic, stress, undrained, void ratio, parameters, model, model parameters.
Fig. 4. (Color) Maximum stress obliquity: Loose and medium dense sands subjected to repetitive fluid pressure cycles to different maximum stress obliquities $\eta_{\text{max}}$. In all four specimens, the pressure cycles begin at $p_0' = 250 \text{ kPa}$ and $\eta_{\text{min}} = 0.20$. Tests end with undrained axial compression from the same initial stress condition at $\eta_{\text{min}} = 0.20$. Notation: $p' = (\sigma_1' + \sigma_3')/2$, $q = (\sigma_1' - \sigma_3')/2$, $\phi_{cs} = \sin^{-1}(\tan \alpha)$, and stress obliquity $\eta = q / p'$. Numbers in square brackets [#] indicate the Test number in Table 2.

Fig. 5. (Color) Confining effective stress: Medium dense sand specimens subjected to repetitive fluid pressure cycles between $\eta_{\text{min}} = 0.20$ and $\eta_{\text{max}} = 0.50$. Tests end with undrained axial compression from the same obliquity $\eta_{\text{min}} = 0.20$. Figure 2 shows all stress paths in detail. Notation: $p' = (\sigma_1' + \sigma_3')/2$, $q = (\sigma_1' - \sigma_3')/2$, $\phi_{cs} = \sin^{-1}(\tan \alpha)$, and stress obliquity $\eta = q / p'$. Numbers in square brackets [#] indicate the Test number in Table 2.
exhibit a shakedown response regardless of their initial density. For reference, the obliquity at critical state is $\eta_{cs} = 0.52$.

* The dense specimen subjected to pressure cycles above the critical state ($\eta_{max} = 0.56$) shows a ratcheting response ($d = 4 \times 10^{-4}$). This specimen gradually dilates during pressure cycles. Hence the frictional resistance evolves from $\phi_{peak}$ towards $\phi_{e}$; eventually, a pressure cycle above critical state obliquity will cause the soil to fail.

* Overall, the initial packing density determines the different failure modes when soils are subjected to pressure fluctuations. Loose soil will contract. Dense soil will experience dilation or shear deformation modes when soils are subjected to pressure fluctuations.

These observations indicate that shear strain accumulation is a function of the initial void ratio $e_o$, the initial confinement $p_o$, and the maximum stress obliquity $\eta_{max}$ reached during pressure cycles.

**Volume Change**

**Void Ratio**

Fig. 7 presents the evolution of the void ratio with the number of cycles for all specimens where pressure cycles start at $p_o = 250$ kPa. Specimens in Fig. 7 have distinct initial void ratios $e_o$ (from $e_o = 0.59$ to $e_o = 0.71$; for reference, $e_{min} = 0.533$ and $e_{max} = 0.786$) and reach different maximum stress obliquity values $\eta_{max}$. The highest rate of change in void ratio occurs during earlier pressure cycles and is more pronounced as the maximum stress obliquity increases. The void ratio $e_i$ measured at the end of the $i_{th}$ cycle evolves towards an asymptotic terminal void ratio $e_T$ in all specimens. The following accumulation model properly fits all datasets (Park and Santamarina 2019):

$$e_i = e_f + (e_o - e_f) \left[1 + \left(\frac{i}{N^*}\right)^m\right]^{-1} \text{ for } m > 0 \quad (2)$$

where the $m$-exponent varies between $m = 0.8$ to 1.0. The model parameter $N^*$ is the number of cycles required for a given specimen to reach half of the asymptotic volume change $(e_o - e_f)/2$. Table 2 lists fitted model parameters for all tests.

**Discussion**

**Particle-Scale Deformation Mechanisms: Threshold Strain**

In the absence of grain crushing, particle-scale deformation mechanisms relate to the strain level $\gamma$ the soil experiences. There are two threshold strains under monotonic loading conditions:

1. all deformations take place at contacts until the elastic threshold strain $\gamma \leq \gamma_{th}^{el}$ that is selected at $G/G_{max} \approx 0.99$, and
2. there are minimal fabric changes until the volumetric threshold strain $\gamma \leq \gamma_{th}^{vol}$. Typically, $\gamma_{th}^{vol} \approx 30 \gamma_{th}^{el}$ (Sands: Vucetic 1994, Ishihara 1996, Clays: Díaz-Rodríguez and Santamarina 2001).

![Fig. 6. (Color) Shear deformation: Cumulative shear strain $\gamma$ versus number of cycles: (a) loose and medium dense specimens; and (b) dense specimens. The initial mean effective stress $p'_o = 250$ kPa and minimum stress obliquity is $\eta_{min} = 0.20$ in all tests. The maximum stress obliquity $\eta_{max}$ is indicated in each case. Dotted lines: Shear strain accumulation model fitted to test results [Eq. (1), Table 2 summarizes model parameters]. The obliquity at critical state is $\eta_{cs} = 0.52$. Numbers in square brackets [#] indicate the Test number in Table 2.](Image)

![Fig. 7. (Color) Volume change: Void ratio versus number of cycles for loose, medium, and dense specimens subjected to fluid pressure oscillations. The initial mean effective stress $p'_o = 250$ kPa and minimum stress obliquity is $\eta_{min} = 0.20$ is common to all tests. The maximum stress obliquity $\eta_{max}$ is indicated in each case. Dotted lines: void ratio evolution model fitted to test results [Eq. (2), Table 2 summarizes model parameters]. The obliquity at critical state is $\eta_{cs} = 0.52$. Numbers in square brackets [#] indicate the Test number in Table 2.](Image)
Slip-down, grain roll-over, and high frictional losses take place at strains above the volumetric threshold (Ishihara 1996; Mueth et al. 2000).

Let’s consider three spherical particles arranged in a triangular configuration and subjected to a normal force $N$ [Fig. 8(a), inset]. The shear force $T$ increases until the contact force $F_{13}$ between particles $\Theta$ and $\varnothing$ becomes $F_{13} = 0$, which indicates contact loss. The extension of the 1 and 3 contact and the contraction of the 2 and 3 contact follow Hertzian behavior. Then, the horizontal displacement $\delta_{h}$ of the top particle $\Theta$ relative to the interlayer height $d \cdot \cos 30^\circ$ yields the equivalent shear strain for contact loss $\gamma_{\text{th}}$ as a function of the mineral shear modulus $G_{\text{n}}$ and the applied confining stress $\sigma'$ estimated from the applied force as $\sigma' \propto N/d^2$ (Santamarina et al. 2001)

$$\gamma_{\text{th}} \approx 1.3 \left( \frac{\sigma'}{G_{\text{n}}} \right)^{2/3}$$  \hspace{1cm} (3)

This analysis anticipates that the threshold strain at contact loss increases with confining stress $\sigma'$ in agreement with experimental evidence (Dyvik et al. 1984; Kim et al. 1991; Vucetic 1994). The threshold strain estimated using Eq. (3) is $\gamma_{\text{th}} \approx 5 \times 10^{-4}$ at $p' = 250$ kPa (Table 1; see data in Silver and Seed 1971; Dobry et al. 1982; Vucetic 1994; Santamarina and Shin 2009).

Clearly, there can be no volumetric strain accumulation when the cyclic stress level is too low for contact loss and fabric change. But, what is the threshold strain for repetitive pressure cycles? Let us compute the incremental volumetric strain in a given cycle $\Delta \varepsilon_{\text{vol}}$ as a function of the change in void ratio between two consecutive cycles $i$ and $i + 1$ (taken at the same fluid pressure at the end of each cycle)

$$\Delta \varepsilon_{\text{vol}} = \frac{\varepsilon_j - \varepsilon_{i+1}}{1 + \varepsilon_j}$$  \hspace{1cm} (4)

Fig. 8(a) shows the absolute value of the incremental volumetric strain $\Delta \varepsilon_{\text{vol}}$, for contractive and dilative specimens plotted against the peak-to-peak vertical strain $\varepsilon_{z \text{pp}}$ for all cycles. Data trends show that (1) volumetric changes diminish as the number of pressure cycles increases, and (2) volumetric changes vanish $\Delta \varepsilon_{\text{vol}} \rightarrow 0$ as the peak-to-peak vertical strain $\varepsilon_{z \text{pp}} \rightarrow 2.0 \times 10^{-4}$.

**Shakedown or Ratcheting?**

The initial state of stress ($p'_0$, $q_0$) and void ratio $e_o$ together with the amplitude of pressure cycles $\Delta u_s$ and the maximum stress obliquity $\eta_{\text{max}}$ determine the shear strain response of a soil subjected to repetitive changes in pore water pressure under constant deviatoric stress. The incremental shear strain $\Delta \gamma_{\text{ss}}$ between two consecutive cycles $i$ and $i + 1$ scales with the maximum stress obliquity when $\eta_{\text{max}} < 0.95 \cdot \eta_{\text{crt}}$, and gradually diminishes towards shakedown [Figs. 6(a) and 8(b)]. Ratcheting takes place when the maximum stress obliquity approaches or exceeds $\eta_{\text{max}} \rightarrow \eta_{\text{crt}}$ [Figs. 6(b) and 8(b)]. Note that Wu et al. 2017 report the onset of ratcheting behavior at $\eta = 0.50$, i.e., close to failure.

**Minimum Volumetric Strain**

The volumetric strain $\varepsilon_{\text{vol}} = \Delta u/B_{\text{max}}$ computed using the small-strain maximum skeletal bulk stiffness $B_{\text{max}}$ provides a lower bound estimate of the volumetric strain the soil will experience during a given pressure cycle $\Delta u_s$. The maximum skeletal bulk stiffness can be computed from the in situ shear wave velocity $B_{\text{max}} = 2 \cdot (V_s^2 \rho ) (1 + \nu ) / [3 \cdot (1 - 2 \nu )]$, where $\nu$ is the small-strain Poisson’s ratio. For example, consider a KAUST 20/30 specimen subjected to $p'_o = 250$ kPa and $\Delta u_s = 100$ kPa where the shear wave velocity for KAUST 20/30 sand increases with confining stress as $V_s = 89$ m/s ($p'_o/1$ kPa$)^{0.21}$ and the small-strain Poisson’s ratio is $\nu \approx 0.15$ (Note: $e_o \approx 0.65$ in—Table 1). Then, the minimum peak-to-peak volumetric strain is $\varepsilon_{\text{vol}} \approx 6 \times 10^{-4}$.

**Maximum Volumetric Strain**

**Terminal Void Ratio**

Fig. 9(a) compares the initial void ratio $e_o$ and the terminal void ratio $e_f$ for specimens with different $e_o$, $p'_o$, and $\eta_{\text{max}}$ (Note: $p'_o = 250$ kPa for the eight specimens in the dotted box, but symbols are $p'$-shifted to facilitate the visualization). Previous studies have suggested that there is a characteristic “terminal void ratio” for each loading condition (Narsilio and Santamarina 2008). Note that the critical state CS is the terminal state for monotonic shear. Results reported in this study show that loose to medium dense specimens contract to reach terminal void ratios that are denser than CS.
However, very dense specimens only dilate if pressurization causes high stress obliquity $\eta_{\text{max}}$, and may rapidly evolve to failure without reaching a unique terminal state.

**Potential Volume Change: Obliquity**

Let us define the normalized asymptotic volume change $(e_0 - e_T)/(e_0 - e_{\text{min}})$ in terms of the initial void ratio $e_0$ at the beginning of pressure cycles $(i = 0)$, the terminal void ratio $e_T$ $(\rightarrow \infty)$, and the minimum void ratio $e_{\text{min}}$. Results discussed above suggest that the normalized volume change caused by fluid pressure cycles depends on the maximum stress obliquity $\eta_{\text{max}}$ [Fig. 9(b)]. Contractive specimens experience volume change when obliquity exceeds $\eta_{\text{max}} > 0.3$, and it is proportional to $\eta_{\text{max}}$ thereafter. On the other hand, significant volumetric dilation in dense specimens requires a stress obliquity $\eta_{\text{max}}$ greater than the critical state stress obliquity $\eta_{c_s} = 0.52$. The minimum void ratio $e_{\text{min}}$, the void ratio at critical state $e_{c_s}$, and the terminal void ratio $e_T$ for pressure cycles $e_T$ are all “asymptotic states” for a given sand (where $e_{c_s}$ and $e_T$ are initial stress dependent). The preceding results show that terminal void ratios fall below the critical state line between $e_{\text{min}} < e_T < e_{c_s}$. Together, Figs. 7–9 suggest that the balance between internal deformation mechanisms depends on initial stress conditions $p_o'$ and $q_o$, the maximum obliquity $\eta_{\text{max}}$ reached in pressure cycles and the initial void ratio $e_o$.

**Design Guidelines**

The volumetric change $e_T$ associated with the maximum asymptotic change in void ratio $\Delta e = e_o - e_T$ induced by pressure cycles as $i \rightarrow \infty$ is

$$e_T = \frac{e_0 - e_T}{1 + e_0}$$

We cannot propose a definitive approach to estimate the terminal volumetric strain $e_T$ for pressure cycles due to the limited data set available at this time. However, the results in Fig. 9(b) suggest

- The $\mu$-fraction is relatively low (i.e., $\mu \leq 0.3$) and is a function of the maximum stress obliquity $\eta_{\text{max}}$.

**Comparison between Pressure Cycles versus $K_o$-Loading Cycles**

The terminal void ratio evolves to its asymptotic state when the sand is subjected to repetitive vertical loading under zero lateral strain (previously reported in Park and Santamarina 2019). While boundary conditions are very different, both studies show that

- There is a minimum strain required for plastic strain accumulation. The vertical threshold strain in the $K_o$ cell varies in the range of 2 to $7 \times 10^{-4}$, which is similar to estimated values in this study.

- All specimens contract in $K_o$-loading cycles, but not in the pore-water pressure cycles with deviatoric loads (Fig. 7). Yet, the terminal void ratio falls between $e_o > e_T > (0.7 \cdot e_o + 0.3 \cdot e_{\text{min}})$ in both $K_o$-loading and pressure cycle studies.

**Ratio between Horizontal-to-Vertical Plastic Strains**

The shear strain accumulation model $\gamma_i$ [Eq. (1)] and the void ratio evolution model $e_i$ [Eq. (2)] allow us to compute the incremental plastic vertical strain $\Delta e_{pl}^i$ and plastic volumetric strain $\Delta e_{vol}^i$ between two consecutive cycles $i$ and $i + 1$. This approach avoids the inherent error magnification in incremental computations using experimental data

$$\Delta e_{pl}^i = e_{pl}^i \mid_{i+1} - e_{pl}^i \mid_i$$

$$\Delta e_{vol}^i = e_{vol}^i \mid_{i+1} - e_{vol}^i \mid_i$$

For small strains, the ratio $\nu^i$ between the incremental horizontal-to-vertical plastic strains in axisymmetric conditions is

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Fig. 9. (Color) Asymptotic volumetric response: (a) evolution of void ratio for medium dense (= blue) and dense sand (= red) specimens subjected to fluid pressure oscillations at different mean effective stress $p_o'$. Empty symbols show the initial void ratio $e_o$ at the beginning of repetitive pressure cycles, while filled symbols show the terminal void ratio $e_T$. The repetitive changes in pore water pressure begin at $\eta_{\text{max}} = 0.20$ in all specimens shown in this figure. The critical state line CSL is $e_{c_s} = 0.845 - 0.074 \log(p_o')$; and (b) normalized volume change $(e_o - e_T)/(e_o - e_{\text{min}})$ caused by fluid pressure oscillations versus maximum stress obliquity $\eta_{\text{max}}$—Loose, medium dense, and dense sand specimens. Numbers in square brackets [#] indicate the Test number in Table 2.
The terminal void ratio evolves towards an asymptotic terminal void ratio $e_T$ as the number of pressure cycles increases; the rate of change is more pronounced for high stress obliquity $\eta_{\text{max}}$.

- The terminal void ratio for pressure cycles $e_T$ falls below the critical state line. The void ratio at critical state $e_{\text{crit}}$ for the same initial stress $p'_0$, and the minimum void ratio $e_{\text{min}}$ of the sand “bound” the terminal void ratio for pressure cycles $e_{\text{min}} < e_T < e_{\text{crit}}$.

- The terminal void ratios for dilative and contractive specimens do not converge to a single trend.

The terminal change in the void ratio $(e_o - e_T)$ in loose and medium dense sands increases with stress obliquity $\eta_{\text{max}}$ and is a fraction of $(e_o - e_{\text{min}})$; for reference, $(e_o - e_T) \leq 0.3 \cdot (e_o - e_{\text{min}})$ in this study.

Dense dilative sands experience minimal void ratio changes and only dilate when $\eta_{\text{max}}$ approaches the critical state, $\eta_{\text{max}} \geq 0.95 \cdot \eta_c$. Consequently, the frictional resistance evolves from $\phi_{\text{peak}}$ towards $\phi_c$ and soils may fail in shear during subsequent pressure cycles.

**Shear Response**

The shear strain accumulation is more pronounced in earlier cycles, in loose sands, in specimens subjected to higher initial mean stress $p'_0$ and in specimens that experience a higher maximum stress obliquity $\eta_{\text{max}}$.

- The shear deformation may stabilize at shakedown, or continue in ratcheting mode. The maximum stress obliquity $\eta_{\text{max}}$ is the best predictor of shakedown or ratcheting.

- Shakedown should be expected as long as pressure amplitudes keep the stress obliquity below $\eta_{\text{max}} < 0.95 \cdot \eta_c$. Conversely, ratcheting takes place when the maximum stress obliquity approaches or exceeds $\eta_{\text{max}} \geq 0.95 \cdot \eta_c$.

Volumetric and shear strain accumulation during repetitive pressure cycles requires a minimum threshold strain which is estimated to be $\gamma \approx 5 \times 10^{-4}$ in this study. A particle-level analysis of contact loss and published experimental data show that the threshold strain increases with confinement $p'$.  

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**References**


