Global vs Local Strain Measurements in Triaxial Tests – Implications

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ABSTRACT: Accurate stress-strain measurements in triaxial compression tests are critical to derive correct values of stiffness, Poisson’s ratio, and the Biot α-parameter. Yet, inherent biases can produce unrepresentative rock properties. This study investigates the impact of different measurements using strain gauges and LVDTs. A detailed analysis reveals the impact of surface compliance at the interfaces between the specimen and end caps. Tested materials include: standard aluminum, Eagle Ford shale, Berea sandstone, and Jubaila carbonate. Results reveal: 1) Contact deformation adds non-linear behavior to the stress-strain response. 2) Seating effects lower the stiffness computed from cap-to-cap deformation measurements. 3) Strain gauges do not show hysteresis evident in cap-to-cap LVDT systems. 4) Bending due to uneven surfaces and misalignment affect cap-to-cap deformation measurements. 5) Confining pressures improve the contact at the interface and reduce partial slippage. 6) Mounting strain gauges on sleeves is ill-advised. 7) The dynamic modulus is higher than the static modulus. 8) The static and dynamic moduli are sensitive to the imposed axial deviatoric stresses. 9) The estimation of the Biot α-parameter is affected by seating effects. We conclude that specimen–bonded strain gauges are preferred to minimize and possibly avoid any of the above effects for pre-peak strain measurements.

1. INTRODUCTION

Strain measurements are critical for the accurate calculation of stiffness, Poisson’s ratio, and Biot α-parameter. These values are used in analyses and designs in the geotechnical, petroleum and mining sectors. Inherent biases within triaxial tests can obscure and distort measured values and produce unrepresentative rock properties (Dendani et al., 1988).

Measurements using cap-to-cap deformation sensors include seating effects due to the mismatch between the specimen and end caps and non-uniform strains along with the specimen due to restraints at the interface (Baldi et al., 1988). Furthermore, studies which compare static and dynamic moduli often reveal large differences, aggravated at low-stress due to the presence of open discontinuities (Cheng, 1981; Fjaer, 1999; Simmons & Brace, 1965; van Heerden, 1987).

In this study, we conduct axial compression tests with intact cylindrical specimens to compare stress-strain measurements obtained with strain gauges mounted directly on specimens against strains computed from deformation measurements with cap-to-cap and local LVDTs. We use these results to quantitatively address and understand discrepancies in estimated moduli obtained from local and global measurements. On that note, we include a complete experimental dataset and a comprehensive analysis of the different possible errors in rock mechanical measurements.

2. PREVIOUS STUDIES

Local measurements avoid unwanted deformations at the interface between the specimen and end caps. Local measurement sensors and methods include: electro level inclinometer (Jardine et al., 1985; Symes and Burland, 1984), Hall effect (Clayton and Khatrush, 1986), digital image processing (Bhandari, et al., 2012; Macari, et al., 1997; Li, et al., 2016; Parker, 1987), local deformation transducer - LDT (Tatsuoka, 1988), proximity sensors (Hird and Yung, 1989), and fiber Bragg grating - FBG (Xu, 2017). The stiffness computed with local deformation measurements is 15% to 45% higher than the stiffness from cap-to-cap measurements (Ishah et al., 2018; Yimsiri et al., 2005; Kung, 2007; Xu et al., 2014; Xu, 2017; Kumar et al., 2016).

3. EXPERIMENTAL STUDIES

This paper investigates differences between global and local measurements and underlying seating effects. The experimental study consists of three parts.

3.1 Part I

The first experimental study involves a standard aluminum specimen (Table 1). We monitor triaxial compression tests using strain gauges, LVDTs, and ultrasonic wave velocities.
Table 1. Specimen specifications, confining pressures and peak stresses used during the loading/unloading cycles.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Length [mm]**</th>
<th>Diameter [mm]**</th>
<th>L:D ratio</th>
<th>Mass [gr]***</th>
<th>Estimated UCS Pressure [MPa]</th>
<th>Confining Pressure [MPa]</th>
<th>Peak Stress [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>50.230</td>
<td>25.105</td>
<td>2.00:1</td>
<td>67.132</td>
<td>-289*</td>
<td>0, 10, 30, 60</td>
<td>60</td>
</tr>
<tr>
<td>Eagle Ford Shale</td>
<td>50.896</td>
<td>25.435</td>
<td>2.00:1</td>
<td>65.639</td>
<td>130−150</td>
<td>0</td>
<td>58</td>
</tr>
<tr>
<td>Berea Sandstone</td>
<td>51.349</td>
<td>25.385</td>
<td>2.02:1</td>
<td>55.194</td>
<td>62−78</td>
<td>0</td>
<td>35</td>
</tr>
<tr>
<td>Jubaila Carbonate 1</td>
<td>57.841</td>
<td>26.813</td>
<td>2.16:1</td>
<td>78.752</td>
<td>24−48</td>
<td>0</td>
<td>21</td>
</tr>
<tr>
<td>Jubaila Carbonate 2</td>
<td>53.953</td>
<td>26.714</td>
<td>2.02:1</td>
<td>72.989</td>
<td>24−48</td>
<td>0</td>
<td>15</td>
</tr>
</tbody>
</table>

Note: *(ASTM B211, 2012) **Uncertainty in length and diameter are 5 μm. *** Uncertainty in mass is 0.5 mg. The average top and bottom surface roughness for the rock samples is ~10 μm.

3.2. Part II

The second experimental study explores the impact of seating effects by re-polishing the specimen ends (ASTM-D4543, 2008). We use the same aluminum standard previously described. We scan the original and polished surfaces of the specimen and end caps using the NANOVEA optical profilometer Jr25 (lateral resolution of 14 μm and vertical resolution of 0.5 μm).

3.3. Part III

We extend the experimental study to rock specimens using the same test protocols described above. We test Lower Eagle Ford shale (Western Gulf outcrop belt, Texas, USA), the late Upper Jurassic Jubaila carbonate (Riyadh outcrop, Saudi Arabia), and Berea sandstone. However, this paper only present data from Eagle Ford shale specimen. Table 1 shows specimen dimensions, axial stresses, and confining pressures. In addition to the local strain and global deformation measurements, we acquire ultrasonic transit times for all samples during loading/unloading cycles taken every 1 MPa of axial load increment.

4. EXPERIMENTAL RESULTS

4.1. Local strain vs. global deformation: Seating effects (aluminum specimen)

The strain rates \( \dot{\varepsilon}_z = 0.5 \times 10^{-6} \text{ s}^{-1}, 10^{-6} \text{ s}^{-1} \) and \( 2 \times 10^{-6} \text{ s}^{-1} \) does not affect the Young’s modulus (Figure 2). The stress-strain curve using cap-to-cap LVDT shows hysteresis and the computed stiffness is \( E \approx 59 \text{ GPa} \). This value is markedly below the stiffness obtained from the strain gauges, which agrees with the reference Young’s modulus for aluminum \( E \approx 69 \text{ GPa} \) (Callister and Rethwisch, 2007 - Figure 2). Lower values of axial deviatoric stress reveal more pronounced non-linear stress-strain behavior when using cap-to-cap LVDT deformation measurements.
We observe seating effects on polished and original rough specimens. Data gathered with the polished aluminum specimen show small seating effects up to an axial strain of $\sim 10^{-4}$ and axial deviatoric stress of $\sim 5$ MPa (Figure 3a). By contrast, the original rougher surfaces show seating effects up to an axial strain of $\sim 7 \times 10^{-4}$ and axial deviatoric stress of $\sim 17$ MPa (Figure 3b).

Seating effects and hysteresis are avoided when using strain gauges bonded on the specimen (Figure 3a). The Young's modulus derived from cap-to-cap LVDTs is 8.5% lower than from strain gauges even for the polished specimen. Clearly, seating effects have a marked effect on the stress-strain curve.

4.2. Bending effect (aluminum specimen)

Specimen bending is particularly noticeable in cap-to-cap LVDT data during unconfined tests (Figure 3). Computed strain from LVDT-1 deformation measurement is higher than LVDT-2. In contrast, measured strains from the two gauges (SG-1 and SG-2 green lines) plot on top of each other. Therefore, using only one single LVDT would lead to biased results.

4.3. Gauge slippage (aluminum specimen)

The Viton sleeve does not always follow the deformation of the specimen during loading/unloading cycles, as the sleeve may slip along the specimen surface. Then, strain values measured using gauges mounted on the sleeve will be smaller than the specimen strain leading to unreasonably high stiffness (Figure 3b: the estimated Young’s modulus $E \approx 419$ GPa is six times higher than $E \approx 69$ GPa).

Fig. 2. (A) Loading sequence. (B) Stress-strain curves for unpolished aluminum specimen. Green line: strain measurement with gauges. Blue line: strain computed from cap-to-cap LVDT measurements. Both stress-strain curves follow the same loading unloading paths, regardless of the different strain rates.

Fig. 3. Measured deformation between two opposite LVDTs and local strains from opposite strain gauges (SG). (A) Strain gauges mounted directly on the specimen. (B) Strain gauges mounted on the Viton sleeve. Green line: average strain measurement with gauges. Blue line: average strain computed from cap-to-cap LVDT measurements. Aluminum specimen polished (left) and original rough (right). Pink box: seating effects (non-linear behavior).
the standard value of \( E = 69 \) GPa. Higher confining pressure hinders sleeve-specimen slip (Figure 4). Nevertheless, mounting even long strain gauges on top of the sleeves is ill-advised.

Strain gauges fixed directly on the specimen prevent slippage but require properly sealed cables to avoid leakage. In our setup, we pierce a small hole through the Viton sleeve directly at the location where the cables attach to the gauges and fill the hole with both silicon sealant and polyurethane.

### 4.4. Confining pressure (aluminum specimen)

Confining pressure improves the contact between the specimen and end caps and reduces seating effects (Figure 4 – Table 2).

<table>
<thead>
<tr>
<th>Confining Pressure</th>
<th>Young’s Modulus [GPa]</th>
<th>Polished LVDTs</th>
<th>Strain Gauges*</th>
<th>LVDTs</th>
<th>Strain Gauges**</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 MPa</td>
<td>62.95</td>
<td>69.89</td>
<td>59.99</td>
<td>419.47</td>
<td></td>
</tr>
<tr>
<td>10 MPa</td>
<td>63.50</td>
<td>69.58</td>
<td>59.66</td>
<td>137.28</td>
<td></td>
</tr>
<tr>
<td>30 MPa</td>
<td>63.87</td>
<td>69.39</td>
<td>58.69</td>
<td>122.29</td>
<td></td>
</tr>
<tr>
<td>60 MPa</td>
<td>63.81</td>
<td>68.91</td>
<td>58.43</td>
<td>106.21</td>
<td></td>
</tr>
</tbody>
</table>

Note: *Bonded on specimen. ** Mounted on Viton sleeve.

### 4.5. Local LVDTs (aluminum specimen)

We also investigate the benefit of mounting LVDTs on the specimen mid-height with a separation of 26 mm between clamps (Configuration C – Figure 1). The local LVDT system shows an improved Young’s modulus \( E = 69 \) GPa. However, the local LVDTs still show hysteresis probably because of sleeve slippage underneath the clamps. Therefore, specimen–bonded strain gauges are the best option for pre-peak strain measurements (Figure 5).

### 4.6. Surface roughness (aluminum specimen)

The unpolished specimen surface is approximately concave in shape with its circumferential ring ~80 μm higher than the center (Figure 6a). The polished specimen has a 10 μm initial roughness (Figure 7). For comparison,
Fig. 6. Surface topography of the (A) original aluminum specimen surface, and (B) instrument end cap. These plots display elevation gathered with a profilometer. The aluminum surface displays higher topographical values closer to the edge with a height up to 80 μm. By contrast, the end cap shows a convex geometry with a maximum height of ~28 μm.

Fig. 7. Surface topography of original rough (orange line) and polished (blue line) aluminum specimen. Data showed along the aluminum specimen circumferential ring, close to the edge.

The ASTM-D4543 prescribes a maximum tolerable surface roughness of 25 μm. Furthermore, the end caps of our testing system have a convex shape with a height variation of ~28 μm (Figure 6b). The unevenness of the end caps causes seating effects even in the polished specimen (Figure 3a). Furthermore, the aluminum specimen end surfaces deform during consecutive tests due to the non-flat end caps.

5. ANALYSES AND DISCUSSION

5.1. Seating effect

The total deformation measured by a cap-to-cap LVDT $\delta_{total}$ involves three components: the specimen $\delta_{spec}$, end cap $\delta_{cap}$, and the interface $\delta_{cont}$:

$$\delta_{total} = \delta_{spec} + 2\delta_{cap} + 2\delta_{cont}$$  \hspace{1cm} (1)

where,

$$\delta_{spec} = \frac{F \cdot L_{spec}}{E_{spec} \cdot A_{spec}}$$  \hspace{1cm} (2)

$$\delta_{cap} = \frac{F \cdot L_{cap}}{E_{cap} \cdot A_{cap}}$$  \hspace{1cm} (3)

Parameters include the specimen length $L_{spec}$, the cap length to the LVDT’s clamp $L_{cap}$, the applied force $F$, materials Young’s modulus $E$, and the cross-sectional area $A$. We analyze seating effects using Hertzian contact theory (Johnson, 1985). We consider a spherical surface in contact with a flat half-space (Note: the underlying assumption is that the majority of the sphere deformation takes place at the contact - Figure 8). The contact deformation $\delta_{cont}$ predicted by Hertzian contact theory is
\[ \delta_{\text{cont}} = \frac{3}{2} \left( \frac{2F^2}{E_{\text{eff}}^2 R} \right)^{1/3} \]  

(4)

where \( R \) is the spherical radius. From the Pythagorean relation:

\[ (R - X)^2 + \left( \frac{D}{2} \right)^2 = R^2 \]  

(5)

where \( X \) is the profilometer maximum convexity and \( D \) the cap diameter. The effective contact stiffness is

\[ E_{\text{eff}} = \frac{2}{\left( \frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_2} \right)} \]  

(6)

Material 1 corresponds to the Titanium end caps, and material 2 indicates the aluminum specimen.

Figure 9 presents the contribution of the specimen, end caps, and contacts to the total cap-to-cap deformation (parameters listed in Table 3). The cap’s deformation provides the smallest contribution (high stiffness and short length). The contact deformation is equal to the experimental cap-to-cap deformation minus the specimen and end caps deformations. The non-linear contact deformation is pronounced in the low force regime.

The values of convexity \( X \) that match the total analytical deformation range from \( X = 14.5 \) to \( 50 \mu m \). These results emphasize the importance of using strain gauges (local measurements) since seating effects are still apparent for the most polished specimen surface.

| Table 3 Parameters used for Hertzian contact analysis |
|------------------|----------|-----------|
| Parameters       | Value    | Unit      |
| \( E_{\text{spec}} \) | 69,000   | MPa       |
| \( \nu_{\text{spec}} \) | 0.33     | unitless  |
| \( L_{\text{spec}} \) | 50.23    | mm        |
| \( A_{\text{spec}} \) | 494.62   | mm²       |
| \( E_{\text{cap}} \) | 120,000  | MPa       |
| \( \nu_{\text{cap}} \) | 0.37     | unitless  |
| \( L_{\text{cap}} \) | 22.4     | mm        |
| \( A_{\text{cap}} \) | 506.71   | mm²       |
| \( D \)             | 25.4     | mm        |

Figure 10 shows the tangential quasi-static modulus computed every 5 MPa of axial deviatoric stress and the dynamic modulus computed from ultrasonic velocities. The static modulus estimated from cap-to-cap deformations exhibits a higher stress sensitivity due to the non-linear seating effects. The static modulus from local strain measurements and the dynamic modulus follow almost flat lines. The dynamic modulus is higher than the quasi-static modulus due to differences in strain levels and rates.

5.2. Seating effects on the static and dynamic Young’s moduli

Figure 11 presents similar results for the dynamic and static moduli of an Eagle Ford shale specimen. Once again, seating effects have a pronounced effect on stiffness determined from cap-to-cap measurements. The stress sensitivity observed for local measurements as compared to dynamic values suggests distinct effects for mineral bonding, layering and global inhomogeneity on quasi-static and acoustic measurements.
5.3. Impact on Biot’s α-parameter

Biot’s α-parameter relates the applied total stress $\sigma$ and pore fluid pressure $P_p$ to the effective stress $\sigma'$ (Biot, 1941),

$$\sigma' = \sigma - \alpha P_p$$  \hspace{1cm} (7)

Biot’s α-parameter requires careful measurement of the skeleton $K_s$ and the grain $K_m$ bulk moduli:

$$\alpha = 1 - \frac{K_s}{K_m}$$ \hspace{1cm} (8)

Results presented above show that cap-to-cap deformation measurements lead to a lower skeleton bulk modulus $K_s$ due to seating effects and results in higher α-values.

6. CONCLUSIONS

This study investigated seating effects and compared local strain and deformation measurements versus cap-to-cap LVDT systems. Key findings follow: 1) Bending due to uneven surfaces and misalignment affect cap-to-cap deformation measurements. The average values obtained from sensor pairs tend to cancel bending effects. 2) Seating effects lower the stiffness computed from cap-to-cap deformation measurements. 3) Contact deformation adds non-linear behavior to the measured stress-strain response; this is more pronounced at low confinement and axial deviatoric stresses. 4) Hertzian contact guides the analysis of seating effects. 5) Strain gauges do not show hysteresis evident in cap-to-cap LVDT systems. 6) Higher confining pressures improve the contact at the interface and reduce partial slippage between the sample and the sleeve; yet, mounting strain gauges on sleeves is ill-advised. 7) The dynamic modulus is higher than the static modulus (local strain measurements) probably due to differences in strains and strain rates. 8) The static and dynamic moduli are sensitive to the imposed axial deviatoric stresses. 9) The estimation of the Biot α-parameter is affected by seating effects. Ultimately, specimen–bonded strain gauges are preferred to minimize and possibly avoid any of the above effects for pre-peak strain measurements.

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8. REFERENCES


