a provision was made to treat the zone in the future for excessive foundation pressures; consequently, extreme changes in pore water pressure and flow were anticipated. The ground surface installation, as suggested by Kinner and Dugan, was found unsuitable; however, the alternate design discussed by them is considered an excellent method for the given field conditions.

It is interesting to note that Reyes has successfully used a device similar to the writer's in similar field conditions. The writer's modified equipment, with the previously recommended pressure loading chamber, is given in Fig. 9 (a modified version of Fig. 2). It is hoped that the equipment will prove suitable for installing piezometers under most artesian conditions.

New Subgrade Model Applied to Mat Foundations[a]

Discussion by Arnold D. Kerr[2]

A review of the graphs presented reveals that the mat-soil contact pressures in Figs. 12–15 do not satisfy vertical equilibrium. This condition requires that in each of these figures the pressure curves enclose the same area, but they don't appear to do this. The author may want to clarify this matter.

Discussion by J. C. Santamarina,[3] Student M. ASCE and C. W. Schwartz,[4] A. M. ASCE

Horvath presents an interesting analysis for mat foundations, that offers a compromise between the simple but sometimes inaccurate Winkler approach and more rigorous solutions that are either limited to simple geometries and material properties or costly and time-consuming to apply. However, one disturbing trend in the parametric studies presented by the author is the lack of equilibrium between the applied loads and the resultant of the contact pressures calculated from the RSC solution. This is most clearly illustrated by the contact pressure distributions in Figs. 13 and 14. In general, the contact pressure resultants differed from the applied loads by 5–15% in the parametric studies. The discussers suspect that these discrepancies are due to the treatment of the mat edge boundary in the finite difference solution. The Winkler solution can be readily corrected at the boundary by using an effective subgrade reaction coefficient of $k' = k/2$ at the boundary node (e.g., see Ref. 27). The corresponding boundary correction for the RSC model is not so clear, however, because the edge node is now coupled to its neighbors (12). In this case, it appears that the simplest way to minimize the equilibrium violation is to use a very close node spacing.

The discussers have attempted to determine the influence of node spacing on overall equilibrium by programming the RSC finite difference solution on a microcomputer. Unfortunately, attempts to reproduce the author's results were hampered by extremely slow convergence of the iterative equation solver used in the microcomputer solution. The rate of convergence was slowest for very stiff mats, and the cause of the problem again appears to be the boundary conditions at the edge of the mat. In particular, the finite difference expansions from Ref. 12, when applied to the second node beyond the edge of the mat, produce an equation with a very small diagonal and very large off-diagonal terms. This in turn results in either divergence or very slow convergence of the iterative solution. (The iterative equation solver did work well for the Winkler-type degenrate from of the RSC model, for which the above problems do not arise.) As Horvath used a direct elimination equation solver in his computations, the discussers are curious whether he also encountered any numerical difficulties at these boundary nodes.

The discussers also performed a set of finite element analyses for 12 of the parametric studies reported in Ref. 12 and Figs. 10–18. These analyses assumed the same problem geometries and material properties (including the variation of soil properties with depth) used in the original parameteric studies. The finite element discretization used in these analyses was quite fine, with 13 eight-node isoparametric elements along the bottom of the mat. (In the Milovic, et al. solution cited by the author, a relatively coarse mesh with only 5 low-order quadrilateral elements along the bottom of the mat was used.) Some detailed results from our analyses are summarized in Tables 3 and 4. Our overall conclusions from the finite element analysis results are as follows:

1. Qualitatively, the finite element and RSC predictions for the shape of the settlement profile and the distributions of moments and contact pressures are in agreement.
2. The results from the RSC solution are closer to the results from the finite element analyses than are the results from the Winkler solution.
3. The magnitudes of the maximum moments (Table 3) from the two analysis methods are in rough agreement except for the case of distributed loads on flexible mats. However, the magnitudes of the moments are small in these cases and the discrepancies in the predictions are of little practical significance.
4. The center-line contact pressures predicted by the two analysis methods are in very close agreement for all cases studied. However, there are considerable discrepancies in calculated contact pressures at the edges of mats supported on clay foundations. In all of these clay cases, the edge pressures from the RSC solution are less than 50% of

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TABLE 3.—Comparison of Maximum Bending Moments in Mat, Computed from Reissner Simplified Continuum (RSC) and Finite Element (FE) Methods

Maximum Bending Moment (kip-ft)[a]		MAT THICKNESS			
		1 Foot		6 Feet	
Loading condition (1)	Soil type (2)	RSC (3)	FE (4)	RSC (5)	FE (6)
Single-line load	Clay	190.7	169.1	591.3	644.7
	Sand	109.9	98.3	503.3	539.7
Three-line loads	Clay	50.1	44.8	88.2	95.2
	Sand	36.9	31.8	76.9	80.9
Uniform pressure	Clay	11.7	6.5	98.7	79.9
	Sand	4.5	1.8	75.9	52.5

[a]1 kip-ft = 1.36 kN-m; 1 ft = 0.305 m.

TABLE 4.—Comparison of Settlements at Center of Mat, Computed from Reissner Simplified Continuum (RSC) and Finite Element (FE) Methods

Settlement at Center of Mat (ft × 10^{-2})[a]		MAT THICKNESS			
		1 Foot		6 Feet	
Loading condition (1)	Soil type (2)	RSC (3)	FE (4)	RSC (5)	FE (6)
Single-line load	Clay	29.4	16.5	12.8	5.3
	Sand	6.1	5.1	2.3	2.2
Three-line loads	Clay	14.4	5.6	12.3	4.7
	Sand	2.5	2.2	1.9	1.8
Uniform pressure	Clay	14.9	6.0	12.3	4.8
	Sand	2.2	2.0	1.9	1.8

[a]1 ft = 0.305 m.

those calculated from the finite element analysis. (Of course, it is debatable whether any of these high edge stresses will occur in a real soil foundation; local yielding will likely reduce the maximum stress concentrations.)

5. The contact pressures calculated from the finite element analyses are in equilibrium with the applied loads in all cases.

6. The predicted center-line settlements (Table 4) from the two analysis methods are in close agreement for mats on sand foundations; however, the settlements from the RSC analyses are often more than 100% greater than the corresponding finite element results for mats founded on clay.

In general, our finite element results confirm many of the points made in the paper. The RSC solution does provide more accurate values for bending moments and contact pressures than do either the Winkler or conventional analysis methods. However, for some cases—in particular, for stiff mats on undrained clay foundations—the RSC solution appears to underpredict the edge stresses and grossly underestimte the average settlement magnitudes (as the author himself noted). It thus appears that the "paradox of using the Winkler subgrade model to calculate moments and some other analytical procedure to calculate settlements for mats" must also apply to the RSC model, at least in some cases.

In conclusion, the discussers believe that the Winkler subgrade model, despite its limitations and inaccuracies, will remain in common use for simple and quick preliminary calculations. For more refined analyses, the discussers expect that the finite element method, with its great power and versatility, will become the standard, given the increasing availability and decreasing cost of in-house computer facilities. Two-dimensional finite element analyses for the mat foundation problem are within the capabilities of current generation microcomputers, and much of the time and expense of input preparation can be eliminated through the use of simple mesh generator preprocessing routines.

Appendix.—References

27. Bowles, J. E., *Analytical and Computer Methods in Foundation Engineering*, McGraw-Hill Book Co., Inc., New York, N.Y., 1974.
28. Milovic, D. M., Touzot, G., and Tournier, J. P., "Stresses and Displacements in an Elastic Layer Due to Inclined and Eccentric Load over a Rigid Strip," *Geotechnique*, Vol. 20, No. 3, 1970, pp. 231–252.

Closure by John S. Horvath,[5] M. ASCE

Both discussers raise the question of contact-pressure equilibrium. As suspected by Santamarina and Schwartz, the apparent discrepancy arises from the way in which the finite-difference formulation was applied to the mat edge. The writer chose to place a node at the edge, which means that the parameters at this node (mat stiffness, applied load, etc.) are the average of those that exist over a 2-ft distance that extends 1 ft into the mat and 1 ft beyond the mat. Consequently, the calculated contact pressure at the mat edge, though plotted in the figures as though it were a value at a single point, is actually the average value over a 2-ft distance that extends 1 ft beyond the edge of the mat. If this additional pressure is included, the calculated pressures are in much better agreement with equilibrium. In future analyses, the writer would probably place the exterior node just inside of, rather than at, the mat edge. This also eliminates having to average the mat stiffness and load at the edge of the mat, as the writer did in the reported analyses using a procedure recommended in Ref. 9.

The writer did not encounter any of the difficulties with numerical stability at the boundary nodes reported by Santamarina and Schwartz.

Finally, the writer is gratified to see what is, in general, very good agreement between the RSC and the finite element analyses performed by Santamarina and Schwartz. The writer still belives that there is a need in routine mat foundation analysis and design for the RSC, as it can be extended to the more typical three-dimensional problem more readily than finite element analyses.

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