TECHNICAL NOTE

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Centrifuge Modeling: A Study of Similarity

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ABSTRACT: Data are presented of geotechnical centrifuge models of reinforced soil retaining walls in which the effects on wall behavior of differences between prototype and model, boundary effects, stress paths, instrumentation, and model repeatability are studied.

KEYWORDS: centrifuges, models, reinforced soils, scale effects, stress paths

Nomenclature

- b Width of model
- b' Distance from initial strip break to lateral wall
- g Gravitational acceleration of the earth
- m (Subscript) model
- p (Subscript) prototype
- H Height of wall
- K Coefficient of earth pressure
- L Width of the wall
- N Scale of a model
- γ Unit weight of soil
- ϕ Angle of internal friction
- ϕ^* Soil-wall friction
- ψ Dimensionless ratio related to wall failure by strip breakage

Introduction

Geotechnical centrifuge modeling is now a well recognized research technique by which a physical model of soil can be made to satisfy the requirements of similarity closely. In the process of studying reinforced soil wall behavior using centrifuge modeling, additional analyses and tests were performed to investigate modeling conditions and departures from similarity. Fifty-two models of fine, uniformly graded Ottawa sand, reinforced with strips of high strength aluminum foil to form vertical reinforced soil retaining walls were constructed and loaded to failure by increasing self-weight using the centrifuge.

The overall purpose of the study was to examine the influence of changes in the retained fill and the foundation soil on wall behavior [*1,2*]. This technical note presents some of the results obtained in analyzing departures from similarity and assessing model repeatability (grain size effects are discussed by Santamarina in Ref *1*). While the study examined the behavior of reinforced soil prototypes, results of these tests are relevant to the modeling of other systems as well.

Boundary Effects

A danger exists in extrapolating the behavior of small physical models with relatively close boundaries to that of full-scale configurations in which boundaries exist at geometrically greater distances. The model boundaries in such a situation will provide frictional support to the edges of the model, and these boundary effects may lead to unconservative predictions when extrapolated to a prototype with different boundary conditions.

Various authors have commented on this modeling effect. Terzaghi [*3*] recommended that the ratio of the width b to the height H of 1g models should be greater than two and that any measurement should be made at the center of the model. Lyndon and Schofield [*4*] cited research by Fuglsang [*5*] on centrifugal model tests of slopes in boulder clay fills, showing that a b/H ratio between 3 and 4 was necessary to minimize boundary effects. Vargin [*6*] studied the effect of side friction on the pressure against a 1g model wall, and later, Lazebnik and Chernysheva [*7*] used a simple analysis of forces to confirm Vargin's results. More recent three-dimensional theoretical stability analysis [*8,9*] could also be used to shed light on this issue.

The goal of this study was to provide empirical data on boundary effects of these granular reinforced soil retaining walls. Two model heights were used; the ratio b/H of full model width to model wall height was 2.5 for seventeen "tall" models (H = 144 mm), and 4.4 for 17 "short" models (H = 80 mm). Although boundary effects were expected, the first clear evidence of their existence was signaled by the fact that the average ratio of the distance measured from the reinforcing strip, which broke first, to the closest model side boundary b', divided by the wall height H, was 1.0 for the tall models and 1.4 for the short models. Two approaches were used to

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assess the effect on model behavior: (1) reduce friction at the boundaries and (2) measure boundary effects.

The magnitude of the friction between the soil and the side boundary wall is characterized by ϕ^* and depends on parameters, such as the hardness and smoothness of the wall, and on the soil mineralogy, angularity, size, and surface texture. In general ϕ^* for a granular soil is taken to be between $\phi/2$ and $\phi/3$ where ϕ is the angle of internal friction of the soil. In an attempt to reduce ϕ^*, a number of "intervening layer" designs were tested on a modified shear box, at different levels of normal stress. It was observed that (1) lubrication alone in the form of grease, oils, or silicones had little or no effect in reducing ϕ^*, which ranged between 22 and 25° for aluminum and plexiglass respectively, and (2) the best intervening layer system consisted of a thin layer of grease applied to the wall and covered with very thin nylon; this system reduced ϕ^* to values ranging between 5 to 9°.

The second approach used to assess the effect of ϕ^* on wall stability was to measure the effect of boundary friction on wall stability by comparing models identical in every way except for width b and ϕ^*. Eight short models were tested for this purpose, with b/H ratios ranging from 0.61 to 4.4. Three of the models were tested with the boundaries prepared with the grease-nylon friction reducing treatment, and the other five without friction reducing treatment.

If no boundary effect existed, then models of all widths should have failed at the same centrifugal acceleration. However, the measured accelerations at failure were lower for the wider models. The data are plotted in Fig. 1. The second horizontal axis on that figure shows percentage error, defined as the ratio $(N_{\text{measured}} - N_{\text{theoretical}})/N_{\text{measured}}$, where $N_{\text{theoretical}}$ corresponds to the ideal case of no boundary effect and was calculated with a method similar to that used by Lazebnik and Chernysheva [7].

From these results it is concluded that for reinforced soil models, a ratio of b/H equal to 4 is desirable to minimize boundary effects; boundary effects increase rapidly for smaller values, and little improvement appears to be derived at greater ratios of b/H. Nonetheless, even in wide models, correction must be made for boundary support; for example, in models with $b/H = 4.4$ and no friction reducing treatment, the estimated overprediction of wall safety is 10%, while in "tall" models with $b/H = 2.5$ the expected error is unconservative by about 20%. It can also be observed that whereas the effect of the friction reducing wall preparation was beneficial, by not eliminating friction completely, correction for boundary effects is still necessary.

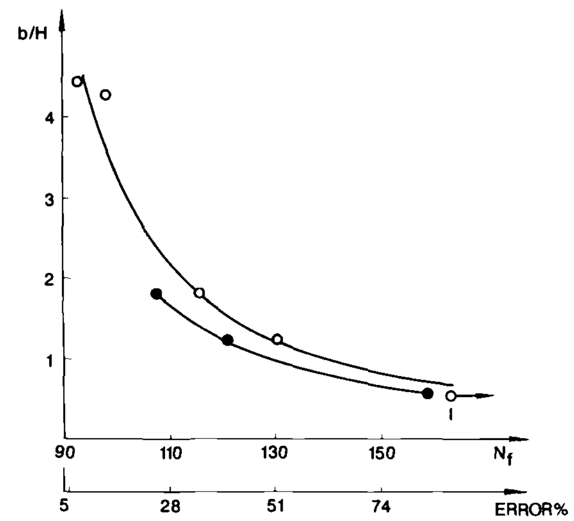

FIG. 1—*Acceleration at failure versus model width height ratio.*

The Loading Path

Modeling the construction procedure in geotechnical centrifuge modeling is a major obstacle to similarity in model and prototype [10]. The standard procedure in centrifuge testing was followed for structures with some structural complexity and failing under self-weight rather than under externally imposed loading. This involved subjecting completed model walls to increasing centrifugal acceleration to simulate increasingly higher walls until failure occurred, which was sudden and catastrophic in these models.

There are two important discrepancies between the loading paths of these models and their hypothetical prototypes. First, all strips exist in the model and experience normal and longitudinal loading from the beginning of the test when construction is assumed to begin, while reinforcing strips in the prototype begin to experience load only after they are placed as construction proceeds. Second, model and prototype loading paths leading to failure differ in the level of shear stresses mobilized along horizontal planes across the wall (Fig. 2). This second discrepancy calls for elaboration.

Let us simplify the problem by neglecting shear along the back of the wall. Then, the horizontal force acting on the prototype wall caused by pressure from the retained fill, when the height of the wall equals that force at failure, is $\gamma K H_p^2/2$; a crude approximation to the average shear stress at the base of the wall caused by that force is $\gamma K H_p^2/(2L_p)$, where γ is the unit weight of the soil, K is some coefficient of earth pressure, L is the width of the wall, and the subscript p refers to the prototype. The corresponding estimation for the average shear stress at the base of the centrifuge model wall is $N\gamma K H_m^2/(2L_m)$ (where N is the scale of the model, and the subscript m refers to the model), which is equal to that of the prototype since $H_m = H_p/N$ and $L_m = L_p/N$. However, if the same analysis is repeated when the height of the prototype wall is half that at failure and the model wall is subjected to an acceleration $N/2$, half that at failure, the average shear stresses at the bases of the walls are not equal. Under these circumstances, the horizontal force acting on the prototype wall is $\gamma K H_p^2/8$ and the average shear stress at the base of the wall, using the same approximation, is $\gamma K H^2/(8L_p)$. For the case of the model, the horizontal shear stress at the base of the wall is $N\gamma K H_m^2/(4L_m)$, which is equal to $\gamma K H_p^2/(4L_p)$, or twice that of the prototype.

The important effects of this discrepancy on wall deformation were demonstrated by simple direct shear tests on unreinforced sand. The loading paths in these tests simulated the stresses experienced by an element of soil at the base of a 10-m high reinforced soil wall "under construction" in ten load increments in the model and the prototype using the same simplifications made above. The two stress paths and the resulting deformations are plotted in Fig. 3. It is clear from the plot that even though the stresses at the ends of the tests are equal, the stress path of the model caused a deformation three times larger than that caused by the stress path of the prototype. Thus, aside from the unrealistic loading of individual

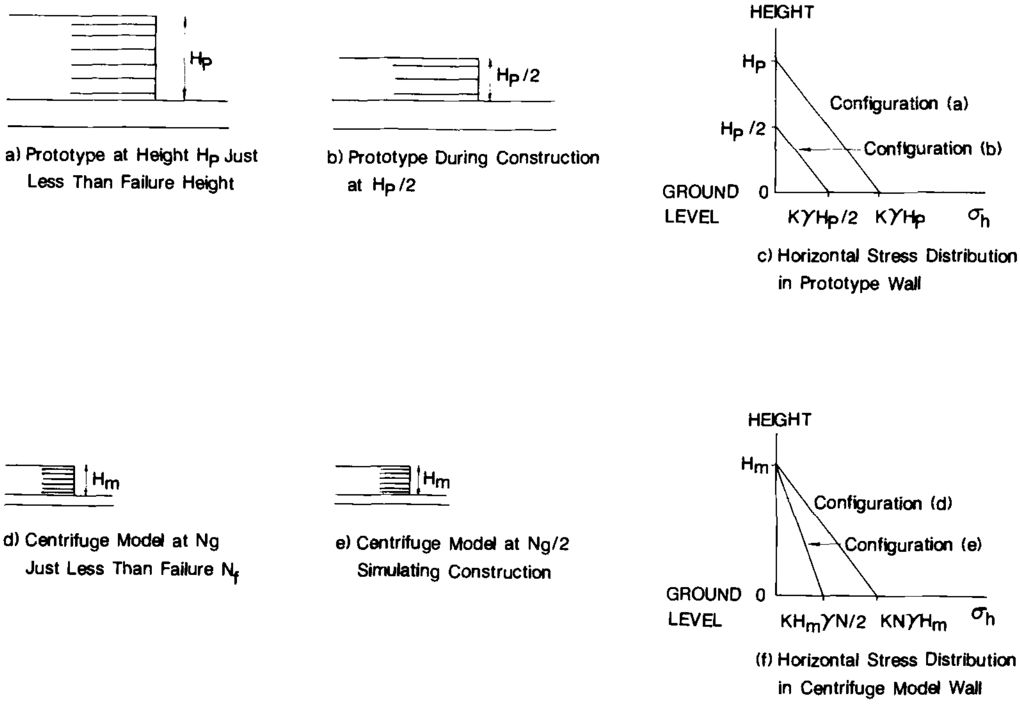

FIG. 2—*Comparison of prototype and model horizontal stresses.*

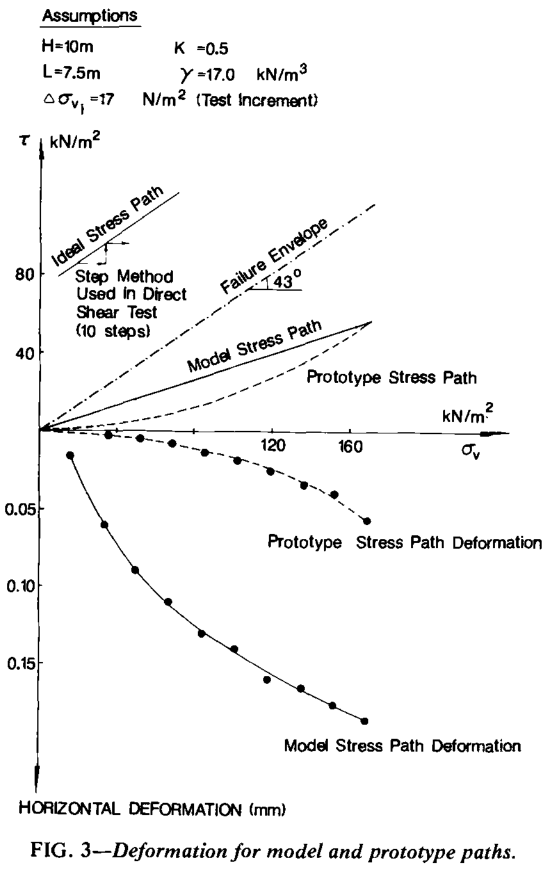

FIG. 3—*Deformation for model and prototype paths.*

strips and its potential effects on internal stability, the response of model walls will be to exhibit significantly more forward tilt than similar prototypes. This effect was evident in the centrifuge model walls that tilted forward between 6 and 17% before failure, in contrast to prototype walls that are seen to tilt forward between 1 and 5%.

Effects of Instrumentation

There was concern during the study that the weight of the plungers of the two displacement transducers (0.26 N at 1 g), which rested on thin bearing plates on the top of the models, positioned at the one third and two thirds points across the width of the wall, might induce premature failure. Assessment of the patterns of failure in 28 models indicated that (1) twice as many models failed first in the vicinity of a transducer than at positions elsewhere in the wall and (2) models with fewer reinforcing strips were more sensitive to the position of transducers with regard to the position of the first strip that failed.

Five models were repeated without transducers to evaluate the effect of the instrumentation on the strength of the walls. A dimensionless parameter ψ that characterizes wall strength was used to compare the results. There was no evidence of the influence of the transducers on the values of ψ at failure.

Model Repeatability

The repeatability of model behavior is important to establish the validity of results, especially in the case of experimental programs with only a few models. Malushitsky [11] concluded that it is necessary to repeat an experiment four times to obtain a stable average result with an accuracy of 7 or 8%, although the study of the effect of a single parameter may be conducted by a series of models with a lower rate of repetition.

For the parametric study in this research, an average rate of repetition of 1.6 was used. But to examine consistency, five identical reinforced soil model walls with sand foundations and sand retained fill, were tested at various times during the study. The model numbers, which indicate the order in the test program, and the corresponding values of ψ at failure are Model 4: 1.367, Model 5: 1.307, Model 35: 1.219, Model 41: 1.366, and Model 52: 1.397. While there is no evidence of a systematic change in ψ as the series continued, Model 35 stands out as having a significantly lower value of ψ than the other models; failure in this model started very close to the boundary ($b'/H = 0.67$) suggesting the existence of a weak reinforcing strip. Nonetheless, the maximum difference in ψ between any two models was 15%; the maximum difference between the overall average (1.331) and ψ of any model, 8.4%, and the maximum difference between the average ψ of any two models and the overall average, 5.1%.

The limited size of the sample does not allow for definitive conclusions. However the data seem to indicate, in agreement with Malushitsky, that three or four models of a single prototype are required to keep the error resulting from variation in model preparation and testing at the same level or below other inherent errors in geotechnical centrifuge modeling [12]. In any case, the number of models to be tested depends on the complexity of the model, the variability of the materials, and the skill of the modeler.

Conclusions

Geotechnical centrifuge modeling is an established research tool that is making its way into engineering design. The following conclusions on similarity conditions and repeatability are drawn from data of reinforced, granular soil retaining walls:

1. Boundary effects were small for models where the ratio of model width to wall height was four or more.
2. Lubrication in the form of grease, oils, and silicones had little effect in reducing side friction in a granular soil model. Other more complex systems were more efficient in reducing side friction, but by not eliminating it altogether, attention to boundary effects was still necessary.
3. The stress path of soil and reinforcements in a prototype as it is constructed will not be identical to the stress path in the model in which construction is simulated by increasing acceleration of the completed model wall. This departure from similarity results in larger horizontal deformation in a model wall than in the corresponding prototype, and in different demands on the reinforcement.
4. The load of the plungers of linear voltage displacement transducers was found to affect the position of the initiation of model wall failure, but did not affect the measured strength of the walls.
5. Repeatability of model behavior is important to validate results, especially in the case of experimental programs with only a few models. Three or four models of the same prototype are needed to reduce the error of the result of these models to the level of other errors in centrifuge modeling.

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