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**DISCUSSION**

# Interpretation of bender element tests

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Bender elements have surfaced as versatile transducers for low-strain testing of soils in a variety of cells and load conditions. However, lack of guidelines leads to different implementations among laboratories. The authors present an interesting evaluation of effective length and travel time determination. The purpose of this discussion is to contribute complementary information, summarizing our experience with bender elements in various short-term and long-term tests (a typical installation and examples of application are described in Fam & Santamarina (1995)).

*Dry and wet.* Dry media are preferred when testing with bender elements (high electrical impedance). Insulation is critical when studying wet or saturated specimens, especially in long-term tests and with saturation fluids with high ionic concentration. Eventually, water reaches the crystals by diffusion. The first sign of short-circuiting is a diminished output. If both transmitting and receiving elements are affected, the wave produces a signal that rides on a charging curve. Fig. 9 shows the typical response for a deteriorated bender element in a saturated silty soil subjected to step and a short pulse excitations. As shown, short pulse excitation reduces the effect of the charging current response observed with the step input, and gives a sharper arrival. These data are very similar to results presented by the authors in their Figs 2 and 4, suggesting faulty insulation as an alternative cause for early energy arrival.

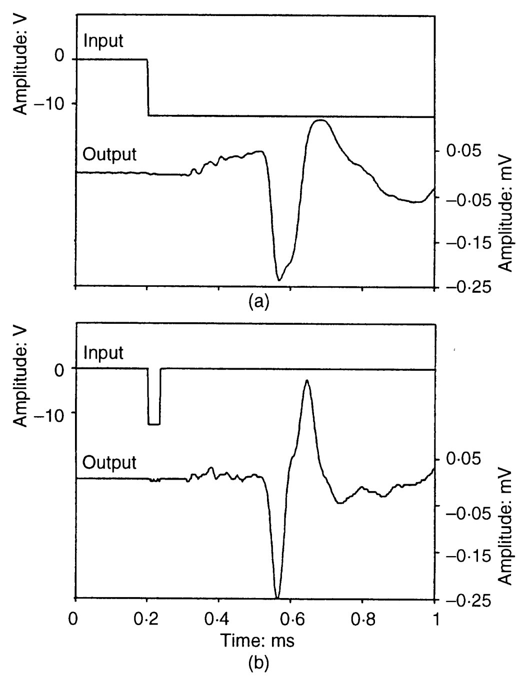


**Fig. 9. Typical response for a bender element with deteriorated insulation in a saturated silty soil, subjected to a step and a short pulse excitation ($\sigma_v'$ = 40 kPa)**

We have tested different coatings; polyurethane and caulking for usage in marine applications were the most efficient and durable (up to 2 months in saturated NaCl aqueous solution). As the short-condition develops, tests can be continued by (1) using a short pulse excitation to facilitate the identification of arrivals, and (2) changing the role of the elements so that an amplified input can be fed to the transducer in the worse condition.

*Reversed polarity.* The main advantage of using wide square functions is to corroborate shear wave arrival by reversed polarity. However, the experimenter must be cautious: P-waves generated at the sides of bender elements (lateral lobes in directivity function) reach the receiver by reflections at cell boundaries and may also show ‘reversed polarity’. This can be readily verified by filling the test cell with water.

*Mechanical analogy.* The input bender element deflects according to the applied voltage, the compliance with the housing and the confinement applied by the surrounding soil. On the other hand, the voltage output produced by the receiving element reflects the time-dependent motion enforced by the surrounding soil. Altogether, the response of the global system is adequately represented by the velocity of a damped, single-degree-of-freedom system subjected to forced vibration on its mass. The dynamic properties of this system can be obtained by standard procedures (impulse or step response, random noise excitation, or frequency sweep).

*Global damping.* Inverted values for the overall damping factor $D$ of the global system vary between $D \approx 0{\cdot}10$ and $D \approx 0{\cdot}50$ depending on mounting, soil type and stress condition. Fig. 10 shows the simulated responses for step and sinusoidal excitation, which closely match corresponding measurements conducted by the writers ($D = 0{\cdot}35$).

The estimated damping ratio characterizes the whole system, including soil, transducers, coupling between the sample and the transducer, and geometric spreading. Hence, tests with transmitter–receiver bender elements cannot be used to study material attenuation. Under certain boundary conditions, it is possible to cancel mathematically the transfer functions for soil–transducer coupling and peripherals by comparing multiple reflections detected with the same transducer (see Fratta & Santamarina, 1995).

*Natural frequency.* The frequency response of the equivalent system is determined by the frequency response of the source element (the receiver element responds to the incoming excitation). Compiled data show that the natural frequency of bender element installations in soil cells varies from $f_n = 2$ kHz to $f_n \geqslant 10$ kHz. Lower values of natural frequency are measured in softer soils and during in-air calibration. Signals may have more than one frequency peak; in low-damping installations, beats are readily identified in the time series.

The bender element usually penetrates about one-third of its length into the soil (about 3 mm to 5 mm). Therefore, one should expect that the change in effective stress will affect the resonant frequency of the crystal–soil system. Indeed, this is the case: Fig. 11 shows the variation of the natural frequency during the consolidation of a very soft bentonite, from $\sigma_v' = 4$ kPa to $\sigma_v' = 100$ kPa. The natural frequency of the response also changes with time in creep tests. De-Alba & Baldwin (1991) completely encapsulated elements in resin, probably disengaging the response of bender elements from the surrounding stress condition; this installation required a battery of transducers in order to induce a measurable response.

*Travel time—signal processing.* Cross-correlation has often been suggested to determine the travel time between two signals, in laboratory and field testing (e.g. Woods, 1978). The determination of travel time by cross-correlation presumes that both signals are of the 'same nature'. If this requirement is not fulfilled, cross-correlation may produce incorrect results. For example, cross-correlation is not warranted in field testing when one of the received signals is the result of multiple travel paths and diffraction effects.

The writers analysed the use of cross-correlation, in the context of the proposed mechanical analogy for cells instrumented with bender elements, assuming a one-cycle sine wave input. Numerical results show that the computed travel

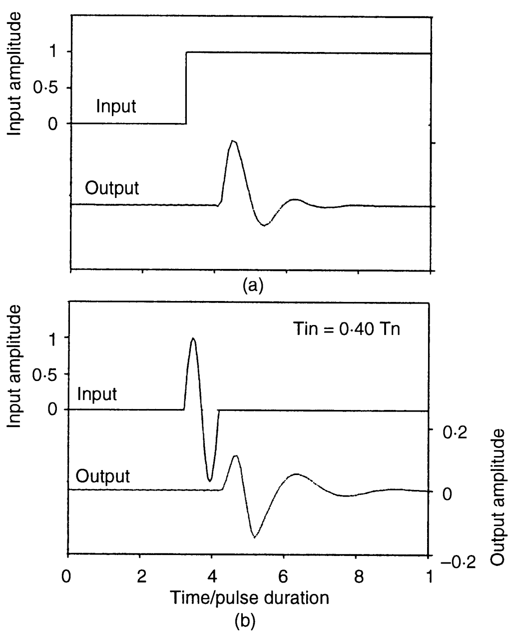


**Fig. 10. Simulated responses for step and sinusoidal excitation—single degree of freedom analogy ($D$ = 0·35)**

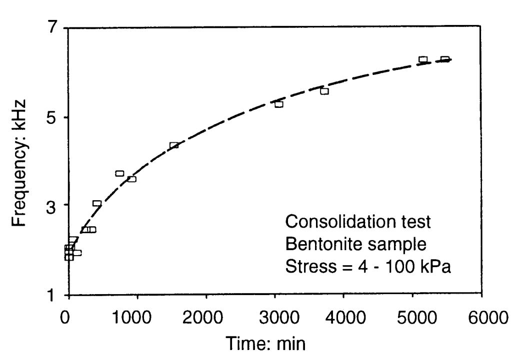


**Fig. 11. Variation of the natural frequency of a bender element installation during the consolidation of a very soft bentonite, from $\sigma_v' = 4$ kPa to $\sigma_v' = 100$ kPa**

time generally differs from the true travel time. Errors depend on the damping ratio, the natural period of the system and the period of the input sine. Table 2 summarizes numerical results; the error in computed travel time is normalized with respect to the natural period of the system. These results show that when cross-correlation is used, the period of the input cycle $T_{in}$ should be equal to or shorter than the natural period of the system $T_n$ and that devices with higher damping ratio $D$ are preferred. In general, travel times are a few $T_n$ long, hence the percentage error with respect to travel time is smaller than the values shown in the table. The use of automated cross-correlation algorithms should be avoided in low-damping systems where the highest cross-correlation peak is approximately $T_n$ away from the real value (first peak).

Table 2. Parametric study of the effect of damping ratio $D$ and input frequency $\omega_{in}$ on the error in travel time using cross-correlation*

| $T_{in}/T_n$ | $\omega_{in}/\omega_n$ | D | | | |
|---|---|---|---|---|---|
| | | 10% | 20% | 30% | 40% |
| 0·5 | 2 | +9·0 | +8·0 | +7·0 | +6·0 |
| 0·8 | 1·25 | +5·8† | +5·1 | +3·8 | +3·3 |
| 1 | 1 | 0·0† | 0·0 | 0·0 | 0·0 |
| 1·25 | 0·8 | −7·2† | −6·8 | −6·2 | −5·8 |
| 2 | 0·5 | −37·0 | −32·0 | −29·0 | −26·0 |

Error is percentage with respect to the natural period of the system: +, cross-correlation time > real time; −, cross-correlation time < real time.

* The second peak in the cross-correlation function is higher. If it is improperly selected the error in the computed travel time is $\approx T_n$.

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The authors have highlighted the importance of correctly identifying the arrival time if accurate values of $G_0$ are to be determined. As Gordon & Clayton (1994) have recently pointed out, the uncertainty that arises from current methods of determining the arrival time severely restricts the reliability and usefulness of bender element measurements of $G_0$. The authors' use of cross-correlation and cross-power spectrum numerical techniques is therefore an important step in the improvement of the accuracy of the measurements by reducing the subjective element of interpretation.

Since the work described in the technical note was completed, research into the use of bender elements of measure $G_0$ for soils and soft rocks has continued at City University, particular attention being given to the correct identification of the arrival time. Jovičić (1995) has successfully developed measurement techniques which effectively remove any subjectivity in the measurements, and from this more recent work the following comments arise.

As discussed by the authors, when a square input wave is used, the initial part of the received waveform (point 0 to point 1 in Fig. 2) represents a near-field wave, which has a component travelling with the velocity of a compression wave. As a square wave is composed of all frequencies, the near-field effect must necessarily be present, complicating the correct determination of the time of the shear wave arrival. For a sinusoidal input wave, Sanchez-Salinero *et al*. (1986) showed that the magnitude of the near-field effect increases the larger is the wavelength relative to the distance between the transmitter and receiver. For the test trace given in Fig. 4 the sample length is only 2·5 wavelengths of the input sine pulse, and a near-field effect is apparent, changing the shape of the received wave with respect to that transmitted. The peak of the cross-correlation or the cross-power spectrum analysis will only strictly identify the travel time of the wave if the shape of the wave remains unchanged, so that the waves must be of predominantly one type, either compression or shear. Mancuso *et al*. (1989) made field measurements comparing the waves received at two points at different distances from the source, so that the two waves would have had similar shapes.

If these techniques are to be applied more accurately, the near-field effect must therefore be minimized. Furthermore, the frequency of the received wave needs to be similar to that transmitted. However, the arrival times of the various peaks and troughs of the sine wave which are given in Table 1 show that the wave had experienced a gentle spreading as it travelled through the sample. Because of this spreading, the travel time $t_a$ measured at the rise point of the wave would be expected to be that closest to the correct arrival of the shear wave; $t_b$ and $t_c$, which are the first peak-to-peak and trough-to-trough measurements, should only be used if $t_a$, $t_b$ and $t_c$ are all equal.

**Authors' reply**

In our technical note we described sources of error in measuring values for the shear modulus $G_0$ relevant to very small strains in bender element tests. The most significant of these was in determining the first arrival of the shear wave at the receiver element.

Theoretical studies by Sanchez-Salinero *et al.* (1986) show that the first recorded arrival may not correspond to the arrival of the shear wave but to the arrival of the so-called near-field component. This is a component of a shear wave which travels with the velocity of a compression wave. It decays rapidly, but for many typical values of shear wave velocity and excitation frequency in bender element tests it may mask the first arrival of the shear wave.

The discussion by Santamarina and Fam describes theoretical studies involving a mechanical analogy for the bender element test and shows results from bender element tests. Unfortunately they have not described the apparatus or soil used in their bender element tests nor have they described their test procedures. Furthermore, they have not given details of their mechanical analogy and it is not clear how the velocity of a damped, single-degree-of-freedom system subject to forced vibration on its mass represents the response of the global system of soil and bender elements. Nevertheless there are helpful contributions made in their discussion.

Santamarina and Fam have drawn attention to the need for careful design and manufacture of bender element installations. Usually, bender element tests on fine-grained soils are carried out on saturated, not dry, samples and to ensure saturation and correct measurement of effective stress the pore pressures will normally be elevated. To prevent electrical short circuits it is essential that the electrical circuits are properly insulated. In the equipment described in our technical note the bender elements, both those made at City University and those purchased from NGI, are encapsulated in epoxy resin and embedded with epoxy resin into slots cut into the platens of a stress path triaxial cell, and the electrical cables are carefully isolated from the metal components of the equipment, the soil, the pore water and the cell fluid, and this is current practice. Unlike De-Alba & Baldwin (1991) we did not find that the response of the bender elements was disengaged from the surrounding stress. Bender elements have now been in regular use for over a decade and, so far as we are aware, perform satisfactorily in tests on saturated soils if carefully installed into the soil testing apparatus. In our research over a period of more than five years, we have found that for a given soil and input signal the form of the received signals remains essentially the same, even for tests on clays lasting several weeks with pore pressures up to 300 kPa. We do not detect any changes in the form of the received signals with time which might have indicated electrical short-circuiting due to faulty insulation. In our experience electrical faults in bender elements are immediate and obvious.

In Fig. 9 Santamarina and Fam show bender element test results which they suggest demonstrate electrical short-circuiting. Fig. 12 shows results of bender element tests on dry Ticino sand (Brignoli & Gotti, 1992) in which both shear and compression bender elements were used simultaneously. These data show that the first arrival recorded by the shear wave bender element at a time of about 300 μs coincides with the arrival of the compression wave. The first arrivals seen by Santamarina and Fam and shown in Fig. 9 at a time of about 0·3 ms (i.e. about 0·1 ms after the input signal) may correspond to the arrivals of the near-field component described by Sanchez-Salinero *et al.* (1986).

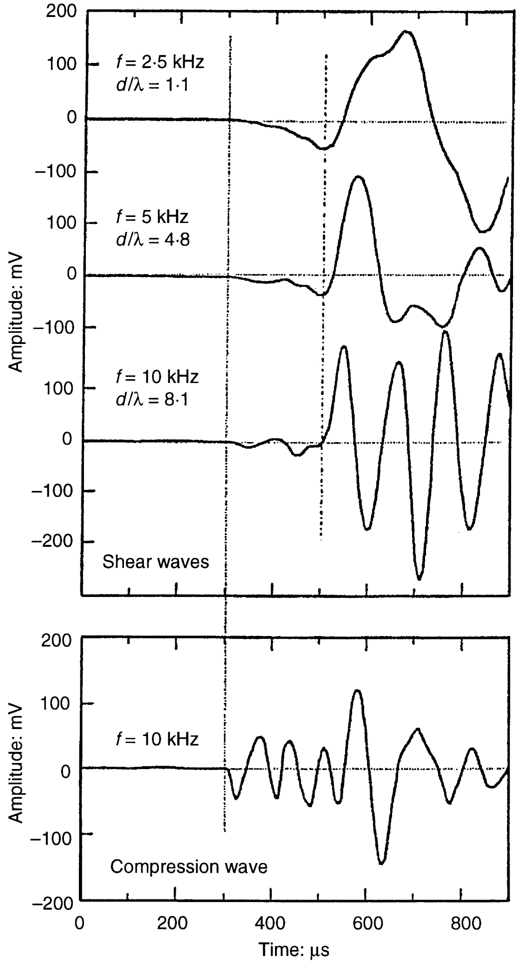


**Fig. 12. Shear and compression waves received by bender elements in tests on dry Ticino sand (after Brignoli & Gotti, 1992)**

Santamarina and Fam suggest that the shear wave arrival can be corroborated by reversing the polarity of the input signal. Sanchez-Salinero *et al.* (1986) showed that reversing the polarity of the input signal reverses the polarity of both the shear wave and the near-field component and so polarity reversal does not positively identify the arrival of the shear wave. Santamarina and Fam do not describe the apparatus in which they carried out their bender element tests: if this was an oedometer cell there may be reflections from the cell boundaries when the cell is filled with water. The tests we described in our technical note were carried out in a triaxial cell with bender elements in the top and bottom platens. With no sample in place but with the cell filled with water no arrivals were recorded at the receiver when the transmitter was activated, demonstrating that there were no travel paths other than through the sample.

We agree with Santamarina and Fam that bender element tests cannot, at present, be used to investigate material damping. Bender element tests provide a simple and reliable method for determining shear wave velocity and hence shear modulus corresponding to very small strains.

In Fig. 11 Santamarina and Fam show the variation of natural frequency of the transmitter element in consolidating bentonite with time. No information is given for the dimensions of the apparatus, the drainage conditions or the location of the transmitter with respect to the drainage boundaries. If, as suggested by the writers, the resonant frequency of a bender element varies with the effective stress in the soil adjacent to the element then the resonant frequency of a bender element located at a draining boundary should stabilize soon after the start of consolidation and the resonant frequency of a bender element located at an undrained face should remain unchanged until the isochrones reach the undrained face.

We agree with Santamarina and Fam and with Jovičić and Coop that determination of travel times by cross-correlation presumes that both signals are of the same nature. We recognized that we were cross-correlating a signal input to one element with a signal output from another and that the input signal to the transmitter may not truly represent the motion of the element. However, using self-monitoring elements (Schulteiss, 1982), we have found that with a single sine pulse like that shown in Fig. 4 the frequency spectrum of the input signal was similar to the frequency spectrum of the signal from the self-monitoring circuit. In Fig. 5, which shows spectra for a standard transmitter element without self-monitoring and for a receiver element, the peaks occur at frequencies of about 5 kHz in both cases. Consequently we do not believe that the cross-correlation analyses in our technical note were subject to significant error.

In our technical note we concluded that, for simple analyses using a single pulse input, the first arrival of the shear wave can be taken as the point of the first inversion of the received signal. Recent work by Jovičić *et al*. (1995) has shown that simpler and more accurate measurements of shear wave travel times in bender element tests can be made using continuous sine wave excitation by tuning the input signal either to remove the near-field component or to achieve forced oscillation of the receiver element at one of its resonant frequencies. With these techniques peak-to-peak or trough-to-trough times may be used without the need for complex numerical analyses.

Engineers are increasingly recognizing the importance of $G_0$ for characterizing soil stiffness and as a basic parameter relevant to non-linear analyses. From our work, from the work described in the discussion to our technical note and from work by others it is evident that bender element tests, applied and interpreted correctly, provide a simple, reliable and practical method for measuring $G_0$ in laboratory tests.

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