Role of Particle Angularity on the Mechanical Behavior of Granular Mixtures

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Abstract: Particle shape affects the mechanical behavior of soils, including packing density, stiffness, volume change during shear, and strength. Laboratory experiments conducted to study the mechanical response of sand mixtures made of round and angular grains show an increase in void ratio, small strain shear modulus G_{max} (constant fabric), oedometric compressibility C_C (fabric changes), and friction angle but a decrease in lateral stress coefficient k_0 as the mass fraction of angular particles increases. These results reflect variations in particle mobility and highlight the relative role of contact stiffness versus fabric changes. **DOI: 10.1061/(ASCE)GT.1943-5606.0000768.**

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Introduction

Previous studies have explored the origin of particle shape (Cho et al. 2006; Santamarina and Cho 2004), the effect of particle shape on the macroscale mechanical response (eccentricity: Rothenburg and Bathurst 1992; Ting et al. 1993; Aloufi and Santamarina 1995; Guo and Stolle 2006; angularity: Cho et al. 2006; Das et al. 2011; roughness: Santamarina and Cascante 1998; Yimsiri and Soga 1999), and engineering implications (early study by Holubec and D'Appolonia 1972). Results showed that all structural and mechanical properties such as void ratio, stiffness, and friction angle are strongly affected by particle shape characteristics such as sphericity, roundness, and smoothness.

Sediments are granular mixtures made of particles of different size and shape. The behavior of these mixtures is quite complex and may not always be estimated as a linear mass average of the properties of the components. The purpose of this study is to investigate the effect of particle shape on the mechanical properties of sands made of mixtures of angular and round particles.

Experimental Study

The experimental methodology is described first, followed by a summary of experimental results.

Materials and Procedures

Material and Mixtures

Mixtures of round Ottawa 20–30 sand and angular blasting sand were prepared at selected mass fractions. These sands have the same mean size ($D_{50} = 0.7$ mm) and a low coefficient of uniformity ($C_u = 1.2-1.9$) but different particle shapes (Ottawa 20–30: roundness = 0.9 and sphericity = 0.9; blasting sand: roundness = 0.3 and sphericity = 0.55; from Krumbein and Sloss 1963). A total of six mixtures were prepared with different mass fractions of angular blasting sand: $F_{ang} = 0$, 20, 40, 60, 80, and 100%. All tests were repeated three to five times to assess variability in the dataset.

Devices

Horizontal stress and vertical deformation were measured in a soft oedometer cell, and the lateral stress coefficient k_0 and compression index C_C (cell: aluminum; thickness of 0.13 mm; inner diameter = 66 mm) were computed. Strain gauges were located at mid-height of the sample, as detailed in Shin and Santamarina (2009). The vertical stress range is limited to $\sigma_v \approx 50$ kPa to keep the circumferential strain $<10^{-5}$ so the error in k_0 is less than 0.01 (Okochi and Tatsuoka 1984). The bottom and top plates included bender elements to generate and detect S-waves (vertical propagation, horizontally polarized).

Procedure

The homogeneous mix was scooped into a small funnel (tube inner diameter = 4 mm) and slowly rained into the oedometer cell from a height of ~30 mm while steadily moving the funnel parallel to the sediment surface. The complementary angle of repose measurements were determined using the same sample preparation method but in a transparent prismatic box (procedure in Santamarina and Cho 2001; detailed discussion in Chik and Vallejo 2005).

Experimental Results

Initial Void Ratio

Plotted data correspond to the void ratio measured at the beginning of each oedometric test [Fig. 1(a)]. The rectangular regions show the $e_{min} - e_{max}$ range for Ottawa ($F_{ang} = 0\%$) and blasting sands ($F_{ang} = 100\%$); the specimen preparation method used in this study produces sands of medium relative density.

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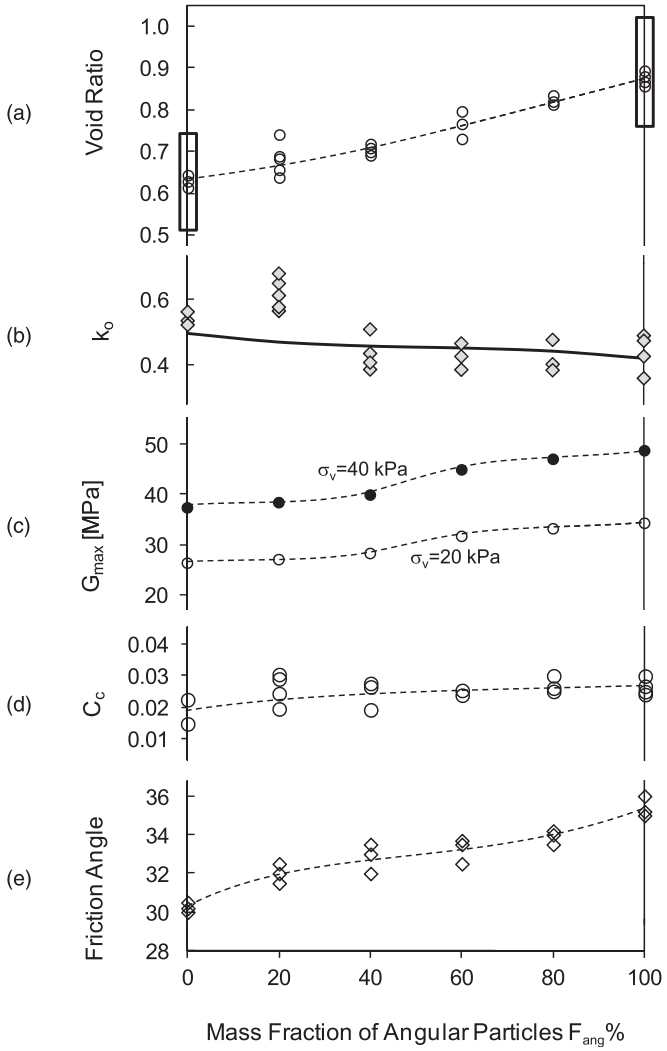

Fig. 1. Experimental results: mixture properties as a function of the mass fraction of angular particles F_{ang}; (a) void ratio; (b) lateral stress coefficient k_0 (the solid line shows Jaky's equation $k_0^{<NC>} = 1 - \sin\phi$); (c) small strain shear modulus G_{max} at 20 and 40 kPa; (d) oedometric compressibility C_c; (e) critical state friction angle ϕ_{cs} based on angle of repose

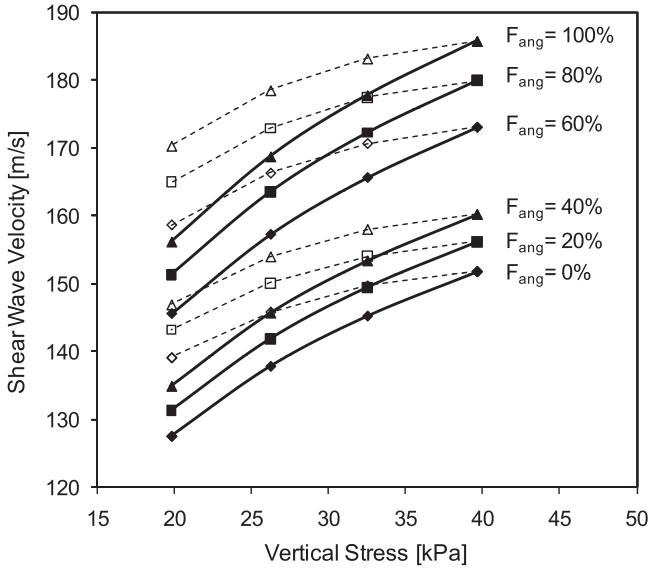

Fig. 2. Shear wave velocity versus effective vertical stress during loading (solid lines) and unloading (dashed lines) for different mixtures of angular and round sands

Stress Ratio at Rest k_0

All specimens exhibited a linear relationship between horizontal σ_h' and vertical σ_v' stresses during the first loading. The stress ratio at rest ($k_0 = \sigma_h'/\sigma_v'$) is plotted for all mixtures in Fig. 1(b).

Shear Wave Velocity

The shear wave velocity V_S was determined at each loading stage during both loading and unloading. Results summarized in Fig. 2 show higher velocity for mixtures with higher fraction of angular particles F_{ang}. However, they all exhibited a similar velocity-stress gradient. All mixtures were stiffer during unloading.

Small-Strain Shear Modulus

The small-strain shear modulus of the soil skeleton ($G_{max} = \rho V_S^2$) was calculated using the measured shear wave velocity V_S and mass density $\rho = \rho_m/(1 + e)$, where ρ_m is the mass density of the mineral that makes the particles. Data summarized in Fig. 1(c) show the range in G_{max} values determined at 20 and 40 kPa.

Compression Index

The change in void ratio Δe versus the effective stress during the first loading was used to compute the compression index $C_C = \Delta e/\log(\sigma_{v-max}/\sigma_{v-min})$ [Fig. 1(d)].

Friction Angle

The angle of repose was considered a proxy for the critical state friction angle and highlights the role of particle mobility during shear [Fig. 1(e)].

Observations

Void ratio e, shear wave velocity V_S, small-strain shear modulus G_{max}, midstrain compressibility C_C and friction angle ϕ increased with the percentage of angular particles; the opposite was true for the stress ratio at rest during the first loading k_0.

Analyses and Discussion

Coefficient of Lateral Stress

The value of k_0 during the first loading was estimated using Jaky's equation (Jaky 1944) and the measured mean angle of repose $k_0^{<NC>} = 1 - \sin\varphi$. The predicted trend was plotted as a continuous line in Fig. 1(b) and showed good agreement with measured k_0 values, except for the mixture with $F_{ang} = 20\%$ angular grains. This result may because of grain mobility and the unique packing configuration often observed in intermediate mixtures (Vallejo 2001).

The value of k_0 increased with increasing overconsolidation ratio (OCR) during unloading. $k_0^{<OC>} = k_0^{<NC>}\,\mathrm{OCR}^m$ was fit to the experimental data, and $m \approx 1.1\sin\varphi$ for all mixtures was obtained (typically, $m = \sin\varphi$; Mayne and Kulhawy 1982).

Velocity-Stress Parameters

A power equation was used to analyze velocity-stress data:

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$$V_S = \alpha\left(\frac{\sigma_v + \sigma_h}{2\,\text{kPa}}\right)^{\beta} = \alpha\left(\frac{1 + k_0}{2}\frac{\sigma_v}{1\,\text{kPa}}\right)^{\beta} \quad (1)$$

where α = shear wave velocity at 1 kPa, and β = sensitivity of V_S to effective stress (i.e., velocity-stress gradient). Empirical parameters α and β are extracted by fitting Eq. (1) to the loading data using the measured values of k_0. As observed in Fig. 2, the β exponent is very similar for all mixtures, and the α factor increases with the percentage of angular particles (Fioravante et al. 1998; Lee et al. 2005).

Trends in Fig. 2 and computed α and β values appear to contradict standard contact mechanics that anticipates stiffer and less stress-sensitive spherical contacts than conical contacts at constant fabric. These results highlight the role of interparticle coordination and the increase in coordination during loading: contact compliance is compensated by the increase in interparticle coordination between two consecutive load steps (Cascante and Santamarina 1996).

Stiffness: Small-Strain Shear Modulus G_{max} versus Oedometric Compressibility

The last observation is in agreement with the higher oedometric compressibility C_C observed in mixtures with a higher fraction of angular particles F_{ang}. The small-strain shear modulus is a constant-fabric measurement of state (instantaneous coordination), whereas oedometric compressibility C_C is a measure of the skeletal rearrangement and changes in coordination at intermediate strains.

Comparison with Micaceous Sands

For reference, grain angularity has a much lower effect on packing density, shear stiffness, and oedometric compressibility than the presence of mica platelets in sands (based on data for micaceous sands reported in Gilboy 1928, Gidigasu 1976, and Lee et al. 2007).

Conclusions

Previous studies showed that particle shape factors (i.e., sphericity, roundness, and smoothness) affect the mechanical behavior of soils, including packing density, stiffness, volume change during shear, and strength. Experiments conducted in this study explored the mechanical behavior of mixtures made of round and angular particles.

The presence of angular particles hinders particle mobility, prevents the formation of densely packed sands, leads to a higher friction angle, and lowers the lateral stress coefficient. For comparison, the presence of mica platelets has a more pronounced effect on sands that the effect of angularity.

Mechanical parameters do not vary linearly with the mass fraction of angular particles. In particular, the highest k_0 value (i.e., the closest one to the isotropic stress condition of $k_0 = 1.0$) is measured for mixtures of ~20% angular particles.

The small strain shear modulus G_{max} and the compressibility of mixtures C_C increase with angularity F_{ang}. This apparent paradox reminds us that the small-strain shear modulus is a constant-fabric measurement of state (instantaneous particle coordination), whereas oedometric compressibility C_C is a measure of skeletal rearrangement and changes in coordination at intermediate strains.

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