Detection of Surface Breaking Cracks in Concrete Members Using Rayleigh Waves

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ABSTRACT

This study examines the use of Rayleigh waves for the detection and sizing of surface-breaking cracks in concrete members. First, finite element simulations are performed to define the conditions for Rayleigh wave propagation in members with rectangular cross-section followed by an experimental study with a concrete beam. Time histories recorded at different locations are 2D Fourier transformed into the frequency-wavenumber domain to enhance interpretation and data analysis. Rayleigh waves form at depths less than half the beam depth. With the introduction of a slot, Rayleigh waves are not observed behind the slot, except for the shortest slot depth, and the slot depth cannot be estimated in the frequency-wavenumber domain. Autospectrum calculations reveal strong Rayleigh wave reflections in front of the slot and by can be used to estimate slot depth when the wavelength is less than half the beam depth.

Introduction

Cracks in concrete and reinforced concrete structural elements reveal adverse effects of applied mechanical loads, thermal loads, shrinkage and environmental deterioration due to corrosion, freeze-thaw and fire. In many reinforced concrete structures, the extent and location of cracking is not easy to detect (*e.g.*, underground pipes, tunnel linings, pavements, nuclear vessels, often also buildings and bridges). Yet, these structures must conform to adequate limits on crack depth and distribution in order to satisfy strength and serviceability criteria.

There are various stress-wave methods for crack and defect detection in concrete members (*e.g.*, ACI, 1998; Lin and Su, 1996; Sansalone and Carino, 1991; Popovics *et al.*, 2000). The ultrasonic through-transmission test is the easiest technique to perform, where measured compression wave velocities are used to identify anomalous regions. Echo methods are also common and use reflected compression waves to measure thickness or to identify flaws (Sansalone and Carino, 1989; Sansalone, 1997; Sansalone and Streett, 1997). The spectral analysis of surface waves (SASW) uses the dispersive properties of Rayleigh waves to identify layering, such as near-surface soils and pavements (Nazarian *et al.*, 1983; Tokimatsu, 1995). The SASW has also been used for flaw detection in concrete elements (Kalinski *et al.*, 1994).

Defect detection in concrete members using acoustic NDT methods is non-trivial. Multiple reflections, mode conversion at interfaces, diffraction healing, and the inherent low-pass filtering effects of concrete render transmission/ echo signals that are often difficult to analyze. Furthermore, studies). On the other hand, Rayleigh waves propagate along the surface of an object with a penetration depth of approximately one wavelength. Other advantages of Rayleigh waves for fracture detection in concrete include high energy content and lower attenuation than the body waves radiated from a surface source. However, 'pure' Rayleigh waves develop when half-space conditions prevail (Rayleigh, 1887), but concrete members have finite dimensions. Furthermore, isolating a Rayleigh wave can be difficult if free boundaries are nearby (Kalinski *et al.*, 1994; Douglas and Eller, 1986). The purpose of this study is to develop adequate measurement, signal processing and analysis procedures for

the information density is lowest near the surface, where the presence of cracks is most prevalent (including tomographic

measurement, signal processing and analysis procedures for detection and sizing of surface-breaking cracks in concrete members, based on Rayleigh waves and receiver arrays. The receiver array (as opposed to just a pair of receivers placed in front of and behind the crack) measurements were used in this study for the following reasons:

- 1) Array data facilitates signal interpretation, detection of multiple modes and reflections.
- 2) Using just two receivers would require the previous knowledge of the location of the crack.
- A Multi-sensor system simulates continuous measurements in field applications such as pavements, tunnel linings beams and lends itself to automation.

This work is an extension of the tests and signal processing, described by Zerwer *et al.* (2002), to detect the size and location of cracks in plexiglas plates simulating slices of beam-type elements. This work also builds on the



Figure 1. The beam cross-section is discretized into 128 linear triangular elements.

ideas presented in the literature concerning Rayleigh wave screening in soils (R.K. Srivastava and N.S.V. Kameswararao, 2002; Kattis *et al.*, 1999; Woods, 1968). In this paper, testing and analysis is done on concrete beams. To begin, a finite element study is done to determine the theoretical limits of Rayleigh wave propagation in a concrete member with a square cross-section. Afterwards, the presented experiments are designed to examine the energy content in a Rayleigh wavefront and to determine the location and depth of a slot in a concrete member.

Rayleigh Wave Propagation in a Beam

Dispersion curves can be analytically computed by obtaining the general solution of the wave equation within the appropriate boundary conditions. Eigenvalue solutions of the frequency equation are used to calculate dispersion curves and define mode shapes for the various propagation modes. Examples of well-known analytical frequency equations are the Rayleigh-Lamb frequency equations for plates and the Pochhammer-Chree frequency equation for rods (Lamb, 1917; Chree, 1889). However, an analytical frequency equation does not exist for a member with rectangular cross-section and four stress-free boundaries. Therefore, in this study, a finite element model is used to calculate dispersion curves and mode shapes for a concrete uncracked beam. The results of these analyses are needed to determine how Rayleigh waves are formed in the tested beams and thus to allow for better interpretation of the experimental results.

Finite Element Formulation

The finite element model used in this study follows the formulation presented by Aalami (1973). This model allows for the generation of dispersion curves for a prismatic bar of arbitrary cross-section. The main assumptions in this model follow:

- a) The geometry of the cross-section remains constant along the length.
- b) The material of the bar is homogeneous, linearly elastic and isotropic (this is an acceptable assumption considering small deformation analysis of an uncracked beam performed herein).
- c) The wave motion is in steady-state and purely elastic.
- d) There is no attenuation within the medium.

The formulation used by Aalami (1973) models wave motion in a three-dimensional bar of arbitrary cross-section as a two-dimensional problem where only the cross-section requires discretization. The cross-section of the concrete member is divided into linear triangular elements in the x-y plane and steady-state wave motion is assumed along the length of the bar in the z direction, as shown in Fig. 1. Stresses and strains within the volume of each element are calculated using the Rayleigh-Ritz energy method (Zienkiewicz, 1971). The derived equation of motion for an assemblage of elements is represented by:

$$G[K]{r} + \rho[M]{\ddot{r}} = 0 \tag{1}$$

where G is the shear modulus, [K] is the global stiffness matrix, [M] is the global mass matrix, ρ is the density and $\{r\}$ are the nodal displacements. Assuming simple harmonic motion, the nodal displacements can be written as:

$$\{\mathbf{r}\} = \{\mathbf{r}_0\} \mathbf{e}^{\mathrm{i}\boldsymbol{\omega} t + \boldsymbol{\phi}} \tag{2}$$

where $\{r_0\}$ are the amplitudes of the nodal values, ω is the circular frequency and ϕ is the phase shift. Substituting Eq. 2 into 1 reduces the steady-state wave propagation problem to an eigenvalue problem:

$$([K] - \Omega^2[M])\{r_0\} = 0 \tag{3}$$

where Ω is the normalized frequency given by:

$$\Omega^2 = \frac{\omega^2 \rho}{G} = \frac{\omega^2}{V_s^2} \tag{4}$$

where V_S is the shear wave velocity. The dispersion curve is calculated by assuming a frequency and calculating the eigenvalues, which in this case are the corresponding wavelengths.

A finite element program was written following this formulation and used to generate the theoretical results presented in this paper. The parameters needed to perform the finite element calculations are the compression wave velocity ($V_p = 4,762$ m/s), the shear wave velocity



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Figure 2. Dispersion curves for the first ten propagating modes in a concrete beam with a square crosssection, calculated from the finite element model.

 $(V_s = 2,820 \text{ m/s})$ and the density ($\rho = 2,400 \text{ kg/m}^3$). The concrete beam dimensions are $1,220 \times 152.4 \times 152.4 \text{ mm}^3$ where only the beam cross-section is discretized (Zerwer, 1999). These values were chosen to match those of the real concrete beam used in the laboratory tests.

The size of the element should be smaller than one tenth the shortest propagating wavelength (Lin and Sansalone, 1992). A study of the convergence rate shows that 100 elements are sufficient to obtain high accuracy for the fundamental modes (Aalami, 1973). In the following model the square beam is discretized into 128 triangular elements, with an average length of 12.5 mm. This choice of element size corresponds to the shortest wavelength of $\lambda = 0.125$ m and translates into a maximum wavenumber of k = 8 m⁻¹ (k = 1/ λ). However, accuracy within 4% of the true values are obtained for the fundamental modes up to a wavenumber of k = 20 m⁻¹, corresponding to $\lambda = 0.05$ m. The error increases for higher modes, where the frequencies and wavenumbers are larger (*i.e.*, shorter wavelengths).



Wavenumber

Figure 3. Rayleigh wave formation in a concrete beam. Mode shapes calculated using finite element model.

Calculated Wave Propagation Modes

The first ten theoretical (from finite element analysis) propagation modes for an uncracked concrete beam are shown in Fig. 2. The dispersion curve for each flexural mode represents two independent modes that are symmetric in the x and y directions. The theoretically calculated modes shown in Fig. 2 will later be compared to the results from the experimental measurements. However, not all the modes shown in Fig. 2 can be observed in experimental results because: a) some of them are not excited by the source used in the testing, b) only vertical accelerations are measured, therefore there is a higher sensitivity to flexural modes rather than to longitudinal or torsional modes, and c) the position of the receiver array with respect to a particular mode shape can dictate whether a mode is observed or not (*i.e.*, nodal lines).

Figure 3 shows the calculated dispersion curves for the first flexural and longitudinal modes. These curves approach the Rayleigh wave velocity as the wavenumber increases, that is, flexural and longitudinal propagation modes turn into Rayleigh wave propagation. Mode shapes for the fundamental flexural and longitudinal modes are

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Figure 4. Source and receiver locations for array measurements. (a) Initial tests, (b) Series I, measurements behind the slot and (c) Series II, measurements straddling the slot.

superposed at wavenumbers equivalent to various beam depths (shown in Fig. 3, boxes 1 to 5). Total values of displacements are plotted in boxes 1 to 5 in Fig. 3. This study shows that Rayleigh wave motion begins when the wavenumber approaches k = 13 (Fig. 3, box 4) and it is fully formed when k > 18 (Fig. 3, box 5). It should be noted that for the beam analysed in this study (cross section of $152.4 \times$ 152.4 mm), k = 13 gives $\lambda = 1/k = 0.077$ m which is approximately half of the beam depth. Also k = 18 gives $\lambda =$ 0.055 m which is approximately equal to 1/3 of the beam depth. The slope of the frequency-wavenumber dispersion curve of the Rayleigh wave portion provides a phase velocity of 2,580 m/s, which correlates with the theoretical Rayleigh phase velocity in a homogeneous half space. These results are then used to interpret the experimental results from the beam testing.

Experimental Measurements

Rayleigh wave measurements are conducted on concrete members with square cross section (beam, 152.4 mm \times

152.4 mm) of the same dimensions as that used in the finite element simulations. The concrete is of normal density, prepared with a maximum aggregate size of 10 mm, and a compressive strength 30 MPa. Wave propagation in the concrete beam is measured using an array of 41 receivers spaced 12.7 mm apart. The source is a 4.76 mm (3/16'')diameter steel bearing dropped from a height of 50 mm (2'')guided by a glass tube of slightly larger diameter. Two accelerometers are connected to corresponding charge amplifiers and an oscilloscope. One accelerometer is used as a trigger mounted slightly behind the source and the other accelerometer records the time histories. The accelerometers are mounted and coupled onto the surface of the beam with beeswax (Fig. 4). The resonant frequency of the accelerometers is 61 kHz. Repeated bounces of the ball bearing are prevented from entering the time histories by limiting the sample length obtained from the oscilliscope. The time histories are not stacked.

Three sets of measurements are discussed:

- In the *initial measurements* the receiver array is moved to different locations on the beam with a constant source location (Fig. 4a). The purpose of these measurements is to verify the presence of the Rayleigh wave mode and to define other propagation modes resulting from the impact source.
- In the *Series I* measurements a slot is cut into the beam (Fig. 4b) and the receiver array is placed behind the slot.
- The *Series II* array measurements are made with the source and receiver positions reversed, so the array crosses the slot (Fig. 4c).

Combined information from Rayleigh wave dispersion (Series I) and energy density (Series II) are used to calculate the slot location and depth. All of the receiver measurements (except for one) are made along the centerline of the beam to avoid measurement of end mode resonances generated by the coupled reflections of the fundamental modes (Hudson, 1943; Oliver, 1957; McNiven, 1961).

Initial Measurements

Four array measurements are made at different locations on the concrete beam as shown in Fig. 4a. The first two array locations are the same with the source moved 101.6 and 304.8 mm away from the intended slot location. The third and fourth measurements are recorded along the top and middle side of the beam respectively. For the last two array measurements the source is located 101.6 mm from the proposed slot locations.

Measurements Opposite the Slot (Series I)

In this set of measurements a diamond saw is used to cut a slot into the concrete beam (4 mm width). The source is placed 101.6 mm in front of the slot and the receiver array is located on the opposite side of the slot with the first Zerwer et al.: Detection of Cracks Using Rayleigh Waves

receiver measurement 25.4 mm behind the slot. The test configuration is shown in Fig. 4b. Array measurements are repeated as the slot is progressively deepened at 12.7 mm (1/2'') intervals up to a depth of 101.6 mm (4'').

Measurements Straddling the Slot (Series II)

For this configuration 20 receiver measurements are made in front of the slot and 21 receiver measurements are located behind the slot. The source is situated 101.6 mm in front of the first receiver measurement. Measurements are repeated as the slot depth is increased by 12.7 mm (1/2'') increments up to a depth of 101.6 mm (4''). The experimental configuration is shown in Fig. 4c.

Signal Processing

Signal processing used for the analysis of experimental results is presented below.

Coherence

Coherence is computed to determine the frequency range where reliable results can be obtained. The coherence between two time series, x(t) and y(t), is calculated as:

$$\gamma^{2}(\omega) = \frac{\overline{G}_{yx}\overline{G}_{yx}^{*}}{\overline{G}_{xx}\overline{G}_{yy}}$$
(5)

where the bar denotes the average of multiple similarly measured signals and * indicates the complex conjugate. The autospectral densities are G_{xx} and G_{yy} and the cross spectral densities are G_{yx} and G_{xy} . Perfect coherence is $\gamma^2 =$ 1 but values above 0.9 are considered to indicate high signal to noise ratio, adequate frequency resolution and linear system response (Santamarina and Fratta, 1998).

The source is activated at a distance of 152 mm (6'') from the middle of the beam. Two sets of twenty measurements are made with the receiving accelerometer mounted 50 mm (2'') on either side of the centerline. Coherence calculations are made between two sets of accelerometer measurements. The average results of 20 individual spectra and coherancies are shown in Fig. 5.

The main energy band for concrete is between 10 and 50 kHz as shown in Fig. 5. Reduced coherence values at lower and higher frequencies suggest the inability of the source to produce either longer or shorter wavelengths and the increase in signal-to-noise ratio.

Spectral Whitening

Spectral whitening balances the amplitudes of the various spectral components, so that low amplitude spectral components are enhanced, whereas high amplitude spectral components are reduced. This provides a uniform distribu-



Figure 5. Power spectral density and coherence measurements using a 4.762 mm steel bearing.

tion of the energy density and facilitates the interpretation of the frequency-wavenumber spectrum. The stretched automatic amplitude adjustment method implemented in this study involves three main steps (Coruh, 1985). The first step is a stretching operation, where the measured signal x(t) is convolved with the stretching function s(t):

$$y(t) = x(t) * s(t)$$
 (7)

The stretching signal s(t) used to separate the different spectral components is:

$$s(t) = sin\left(2\pi f_s t + \frac{\pi (f_e - f_s)}{T_s}t^2\right) \tag{6}$$

where f_s and f_e are the starting and ending frequencies of the stretching function and T_s is the length of the stretching function. The values of f_s and f_e define the range of sinusoids that are represented in the stretching signal.





Figure 6. Effect of spectral whitening on the amplitude spectrum. Originally recorded and whitened signals are shown.

In the second step, the amplitudes of the chosen frequencies are balanced by convolving the stretched signal y(t) with a gain function g(t):

$$\mathbf{y}'(t) = \mathbf{y}(t) * g(t) \tag{8}$$

given that

$$g(t) = L { / \sum_{\tau=t-\frac{L}{2}}^{\tau=t+\frac{L}{2}} |y(\tau)|}$$

where y'(t) is the gain-adjusted signal and L is the window width used to calculate the amplitude factor. Finally, the third step is compressing back the signal by crosscorrelating y'(t) with the stretching signal conjugate $s^*(t)$:

$$z(t) = y'(t) * s * (t)$$
 (9)

The series z(t) is the whitened signal (Coruh, 1985). In this study, each receiver measurement is stretched to 16,384 data

points using a window width L = 20. Selecting a smaller value for L improves the balance of energy density between the spectral components, but significantly increases the computation time. To improve amplitude resolution the stretching function is applied at 10 kHz intervals up to a frequency of 60 kHz. The time history is then reconstructed by superposing the whitened signals for the six frequency intervals. The amplitude spectrum shown in Fig. 6 illustrates the effect of signal whitening. Averaging over short window lengths removes random noise and enhances the coherent signal.

Frequency-Wavenumber Analysis

The array of 41 whitened signals are assembled in a matrix. The two-dimensional Fourier transform is computed, converting time-position measurements into the frequency-wavenumber domain (Peardon, 1986; Alleyne and Cawley, 1991). The magnitude of the complex coefficients is calculated from the Fourier transformed matrix. Contouring the magnitude values produces a plot with a series of peaks that can be used to calculate dispersion curves for the measured propagating modes (Zerwer *et al.*, 2000; Zerwer *et al.*, 2002). The phase velocity for the different propagating modes can be computed from the frequency-wavenumber domain using:

$$V_{phase} = \frac{\lambda}{T} = \frac{f}{k} \tag{10}$$

given that

$$k = \frac{1}{\lambda}$$
 and $\omega = \frac{1}{T}$

The time domain signals contain 1,000 data points, "zero tail padded" to 2,000 points, with a sampling frequency of 1 MHz. The spatial domain is "zero tail padded" to 201 points. To reduce frequency and wavenumber leakage, a Hamming window is applied across the spatial and temporal directions of the matrix. Frequency-wavenumber analysis is performed on measurements where the source and receiver are on opposite sides of the slot.

An example of a frequency-wavenumber plot is shown in Fig. 8a. Overlaid are solid lines representing the flexural modes (from FE analysis), dashed lines for longitudinal modes (FE analysis) and solid dots define peaks (experimental). In addition, the dispersion plots are split into two halves. Positive wavenumbers represent energy moving from left to right through the array, whereas negative wavenumbers define energy propagating in a reverse direction across the array.

Autospectrum Measurements

The autospectral density of a signal provides the energy content within specified frequency intervals. The

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autospectrum of each signal is calculated as follows (Santamarina and Fratta, 1998):

$$|G_{xx}| = \{Re[Z(\omega)]\}^2 + \{Im[Z(\omega)]\}^2$$
(11)

given that G_{xx} is the autospectral density, $Z(\omega)$ is the discrete Fourier transform of the compressed signal z(t). Autospectrum calculations are performed on measurements where the slot bisects the receiver array (test series II).

Experimental Results

Experimental results are presented in a frequencywavenumber space and in terms of autospectra. In the frequency-wavenumber space, the experimental results are shown in the form of isometric lines with dots representing peaks of the Rayleigh waves, and theoretical results are shown as continuous lines representing dispersion curves (equivalent to Fig. 2). In case of the perfect agreement between finite element and test results, the dots should align themselves along the theoretical dispersion curves.

Results from Initial Measurements-Beam Without a Slot

(i) Top Edge measurements—An example of the time domain traces measured at different locations (as in Fig. 4a), on the top surface of the beam without a slot are shown in Fig. 7. Dispersion measurements are shown in Figs. 8a and b. Figure 8a shows measurements done with the source distance of 101.6 mm away from the slot (in this case intended slot, since these measurement are made on an uncracked concrete beam) and Fig. 8b with the distance of 304.5 mm from the slot. Both measurements show a clear Rayleigh wave for wavelengths shorter than half the beam depth, corresponding to a wavenumber of $k = 13.1 \text{ m}^{-1}$ and a frequency of 33 kHz. The shortest wavelength Rayleigh wave is 43 mm (k = 23 1/m) at 60 kHz. The measured peaks of the Rayleigh wave compare well with results calculated from the finite element model (phase velocity of 2,580 m/s). The fundamental longitudinal mode appears considerably weaker than the fundamental flexural mode.

The direct Rayleigh wave (traveling from left to right) becomes less coherent as the source distance increases. The Rayleigh wave is easily identified at the 101.6 mm source distance (Fig. 8a) becoming less distinct at the 304.8 mm distance, as shown in Fig. 8b. Therefore, in the tests with the slot a source distance of 101.6 mm was used. For the reflected Rayleigh waves (traveling right to left), stronger wave energy is recorded for the 304.8 mm source distance than for the 101.6 mm source distance.

Higher vibration modes (modes greater than the fundamental mode) are also recorded in these measurements. Cutoff frequencies are observed at 12.5, 17, 25, 31, 34, and 45 kHz. At low wavenumbers, peaks correspond relatively well to higher mode flexural waves. The higher mode



Figure 7. Time domain traces for accelerometers in locations "1 + 3i" with i = 0 to 13. Measurements on the top surface of the beam, no slot.

vibrations are easily visible at the 101.6 mm source distance, but are less distinctive for the 304.8 mm source distance.

(ii) Side Measurement—Top Array—The Rayleigh wave shown in Fig. 8c is clearly identified, but weaker than in previous measurements. A reflected Rayleigh wave is not observed in these measurements. A few higher flexural modes are also observed.

(iii) Side Measurement—Middle Array—The dispersion measurement made along the middle side of the concrete beam is shown in Fig. 8d. A Rayleigh wave is not measured at this location (a very weak Rayleigh wave is identified for higher frequencies). Peaks do not correspond well with higher flexural or longitudinal modes.

Results From Series I-Beam with a Slot

The results of this test series are presented in the frequency-wavenumber space (Figs. 9a–d). The Rayleigh wave shown in the 2D spectrum in Fig. 9a has lower energy for the initial slot depth of 12.7 mm than in a beam without



Figure 8. Frequency-wavenumber results from initial measurements without a slot. (a) Source distance of 101.6 mm, (b) Source distance of 304.8 mm, (c) Measurement along top-side (101.6 mm distance), (d) Measurement along middle-side (101.6 mm distance).

a slot (Fig. 8a). The wavenumber bandwidth of the Rayleigh wave is now between 12.5 and 18 m⁻¹. Increasing the slot depth to 25.4 mm (Fig. 9b) causes the Rayleigh wave to disappear. Frequency-wavenumber calculations from deeper slots shown in Fig. 9c (50.8 mm) and 8d (76.2 mm) have few peaks and low energy related to Rayleigh waves propagating past the slot. The Rayleigh wave reflected from the end of the beam is weakly observed in these measurements. These measurement establish the presence of

the slot in a beam due to the disappearance of the Rayleigh wave and lower energy corresponding to the peaks in the frequency wavenumber plots (lighter shading indicates less energy).

In addition to the Rayleigh wave, other vibrational modes are observed. A strong direct and reflected first flexural mode (FFM) is detected at wavenumbers less than 5 m⁻¹ (Fig. 9). Higher modes are visible with cutoff frequencies at 12.5, 17, 25, 31, 34 and 45 kHz. These modes



Figure 9. Frequency-wavenumber results from measurements with a slot. (a) 12.7 mm slot depth, (b) 25.4 mm slot depth, (c) 50.8 mm slot depth, (d) 76.2 mm slot depth.

become weaker with increasing slot depth. With the exception of the first flexural mode, higher modes reflected from the end of the beam cannot be accurately identified.

Results From Series II-Beam with a Slot

The results of test series II are presented in terms of auto-spectra. The auto-spectrum for all signals are arranged in space and plotted as shown Fig. 10a for measurements without a slot. In this case relating spectral energy to the Rayleigh wave or other propagating modes cannot be accurately accomplished because several modes may contribute to the energy measured at a particular frequency. Nevertheless, the auto-spectrum calculations can identify disturbances in propagation energy. As shown in Fig. 10b (slot depth 63.5 mm) and 9c (slot depth 76.2 mm, approximately half the beam depth) the slot blocks propagating wave energy, becoming more pronounced as the slot depth increases. The slot generates a reflected Rayleigh wave, which is recorded on receivers to the left of the slot location. The slot depth can be back-calculated using the Rayleigh wave velocity and the frequency cutoff generated by the slot. For example, for the cutoff frequency in Fig. 10b of approximately f = 41.5 kHz and the Rayleigh wave velocity of c = 2,580 m/s, the corresponding wavelength of $\lambda = c/f = 0.062$ m = 62 mm. The slot depth in this case was 63.5 mm. The half slab depth (76.2 mm) corresponds to a frequency of approximately 34 kHz and was calculated from $f = c/\lambda = 2,580/0.0762 = 33,858$ Hz.

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Figure 10. Autospectrum calculations for each receiver measurement. (a) No slot, (b) 63.5 mm slot, (c) 76.2 mm slot, (d) 88.9 mm slot.

Thus, these measurements provide information on the slot depth. However, this approach is only valid for slot depths less than half the beam depth because deeper slots hinder the development of a Rayleigh wave and the results become unclear (Fig. 10d, slot depth larger than half the beam depth).

Discussion and Conclusions

The study presented in the paper shows that by combining information from Rayleigh wave dispersion and

energy dissipation it is possible to determine the location and depth of surface-breaking cracks in concrete beams.

The finite element analysis of wave propagation in an uncracked concrete beam shows that flexural and longitudinal propagation modes turn into Rayleigh wave propagation as the wavelength becomes shorter. The Rayleigh wave does not form at depths greater than half the beam depth because the phase velocities of the fundamental modes diverge at longer wavelengths and energy is spent in flexural modes. An important aspect of Rayleigh wave formation, not specifically discussed, is the effect of beam geometry. This study examines a beam with a square cross-section, as a result the fundamental flexural modes in the x and y directions are identical (*i.e.*, all flexural modes have two identical eigenvalues). If the cross-sectional geometry is rectangular then two different Rayleigh waves can potentially exist in a beam.

Measurements made on a concrete beam without a slot demonstrate that the fundamental flexural and longitudinal modes are excited and a Rayleigh wave is formed at higher frequencies. The measured phase velocities of these modes as well as the appearance of the Rayleigh are confirmed by the theoretical results obtained from the finite element analysis. Additional measurements made on the side of the beam at different depths (top and middle) illustrate that a Rayleigh wave does not exist below half the beam depth. Higher vibration modes are also observed.

Introduction of a slot causes the transmitted Rayleigh wave energy to decrease. The frequency-wavenumber plots show that a Rayleigh wave exists behind a 12.7 mm slot, but disappears behind the 25.4 mm slot. The maximum wavelength, calculated by phase velocity (2,580 m/s) divided by the minimum frequency of pure Rayleigh waves are noticed (approximately 33 kHz), is in the order of 7.5 mm. Thus we can conclude that slots which are much deeper than a dominant Rayleigh wavelength provide an effective screening. However, there are a number of reasons why a Rayleigh wave does not reform behind the slot. First, the source provides more energy in a flexural direction rather than a longitudinal orientation. Second, the Poisson's ratio of concrete is approximately 0.21, further reducing longitudinal mode energy. Third, not enough energy passes the slot to reform the longitudinal mode. Conversely, the reflected Rayleigh wave energy increases.

Autospectrum calculations demonstrate the slot effectively blocks the Rayleigh wave allowing an estimation of slot depth. Although a cutoff frequency is observed for all slot depths, backcalculated slot depths compare well to the real slot depth up to half the beam depth. This last result indicates that energy attenuation due to presence of cracks should be evaluated in future work on signal processing.

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